Acceleration and Deceleration

1. Introduction

The Acc/Dec control method can be classified with an Acc/Dec control before interpolation and an Acc/Dec control after interpolation with respect to the processing order of acceleration and deceleration control.

The Acc/Dec control before interpolation (ADCBI) is constructed differently according to the interpolation type such as linear-, exponential- and S-curve-type interpolation. Because the ADCBI needs to hold a lot of information, related to all the interpolated points, a large amount of memory is required for executing this type of Acc/Dec control. However, because of the large amount of information the Acc/Dec control does not result in machining error because of the increased accuracy.

On the other hand, Acc/Dec control after interpolation is applied in an identical manner for all interpolation methods. Therefore, the implementation is simple but machining errors occur because each axis movement is determined separately. Since Acc/Dec control in ADCAI is individually applied for each axis, acceleration and deceleration for movements of each axis are carried out regardless of the interpolated position. Accordingly, the interpolated points deviate from the desired path. A typical example of this deviation occurs during the corner machining process and the longer the Acc/Dec time, the larger the machining error.

2. Acc/Dec Control After Interpolation

In the case of ADCAI, firstly the NCK, Numerical Control Kernel, interprets a part program using the interpreter module and calculates the displacement distance for each axis, ΔX, ΔY, ΔZ for every interpolation time interval based on the interpreted result using the rough interpolation module. Next, independent Acc/Dec control of each axis is performed with respect to ΔX, ΔY, ΔZ and the fine interpolation then follows. Finally, the total remaining displacement of each axis for every position control time interval is calculated by the position control module.

The Acc/Dec control algorithm of Acc/Dec control after interpolation is different from that of Acc/Dec control before interpolation. Figure 4.1 shows the flowchart for implementing the NCK with Acc/Dec control after interpolation. The big difference with Acc/Dec control before interpolation is that the remaining displacement of each axis is calculated at each interpolation time by rough interpolation and Acc/Dec control of each axis is performed individually. Figure 4.2 shows the change of pulse profile after the Acc/Dec control. It can be seen that the individual pulse profile of each axis is generated by the rough interpolator and the individual acceleration and deceleration scheme is applied to each pulse profile.

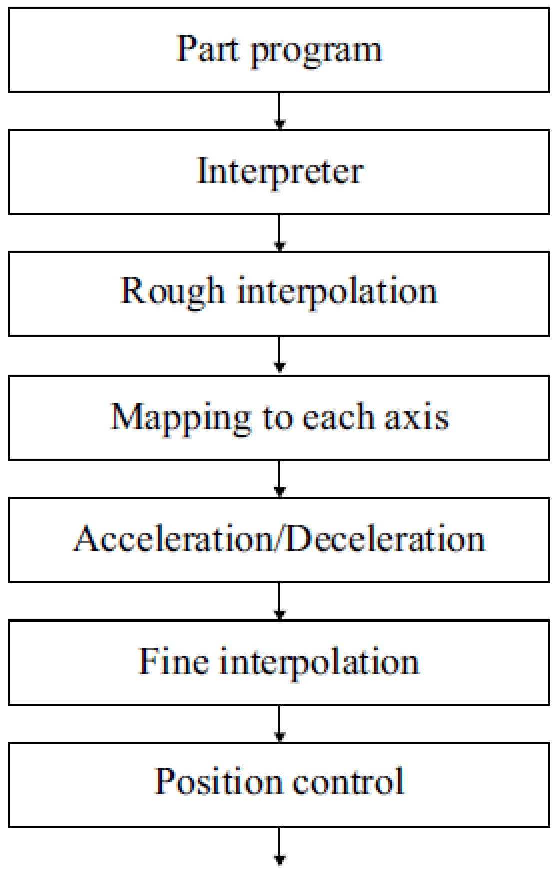

Fig. 4.1 NCK functional procedure with ADCAI

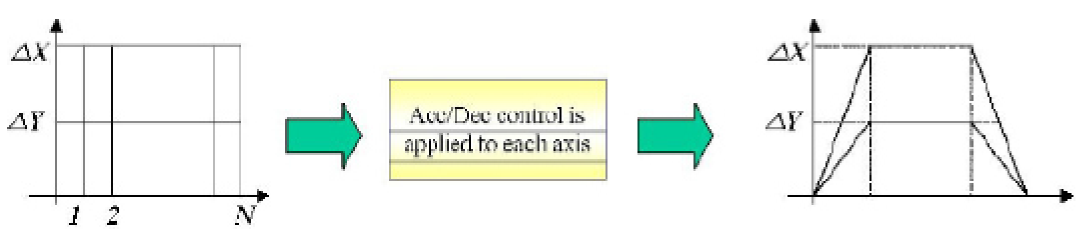

Fig. 4.2 Change to pulse profile after Acc/Dec control

2.1 Acc/Dec Control by Digital Filter

The Acc/Dec control algorithm for the ADCAI method is based on digital filter theory. According to the digital filter theory, if input signal *x*[*n*] is entered into the filter with impulse response *h*[*n*], the output signal *y*[*n*] is represented by the convolution of h[n] and x[n]. **Equation 4.1** shows the general convolution of *f*1[*n*] and *f*2[*n*] for a discrete time system.

$$\begin{aligned} f[n] &= f_1[n] * f_2[n] \\ &= f_1[0]f_2[n] + \ldots + f_1[k]f_2[n-k] + \ldots + f_1[n]f_2[0] \end{aligned} \tag{4.1}$$

$$f[n] = f_1[n] * f_2[n] = \sum_{k=1}^{n} f_1[k] * f_2[n-k] \tag{4.2}$$

As shown in **Fig. 4.3**, if we assume that x[n] is defined as the output of a rough interpolator and h[n] as the impulse response that has the normalized unit summation, as shown in **Eq. 4.3** we can obtain the Acc/Dec pulse profile in which the summation of the input signal is the same as the summation of the output signal after convolution of x[n] and h[n].

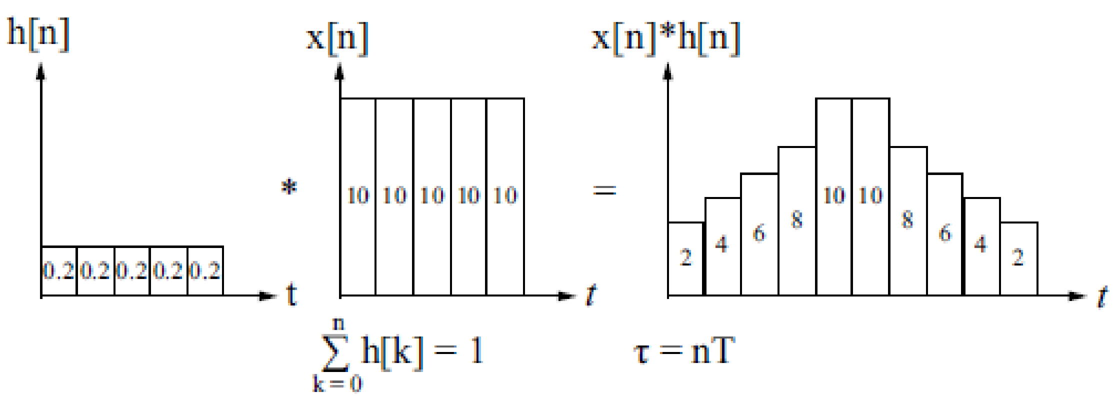

Fig. 4.3 Convolution of rough interpolation and impulse response

Where the Acc/Dec time τ is the multiplication of n and the sampling time T for continuous convolution.

$$\sum_{k=1}^{n} h[k] = 1 \tag{4.3}$$

Further, since h[n] denotes the differentiation of velocity, i.e. acceleration, we can obtain the various Acc/Dec pulse profiles by changes of h[n]. The Linear-type, Exponential-type, and S-curve-type, as shown in **Fig. 4.4**, are used for the Acc/Dec filters of CNC systems. By using various digital filters different output profiles can be obtained even when identical input pulses are used. **Equation 4.4**,

Eq. 4.5, and **Eq. 4.6** represent the Linear-type filter, the Exponential type filter, and S-shape-type filter. Here, T means the sampling time and τ denotes the time constant for Acc/Dec control.

$$H_L(z) = \frac{1}{m}\frac{1-z^{-m}}{1-z^{-1}} \tag{4.4}$$

$$H_E(z) = \frac{1-\alpha}{1-\alpha z^{-1}} \quad \text{where } \alpha = exp^{-\frac{T}{\tau}} \tag{4.5}$$

$$H_S(z) = H_L(z) * H_L(z) = \frac{1}{m}\frac{1-z^{-m}}{1-z^{-1}} * \frac{1}{m}\frac{1-z^{-m}}{1-z^{-1}} \tag{4.6}$$

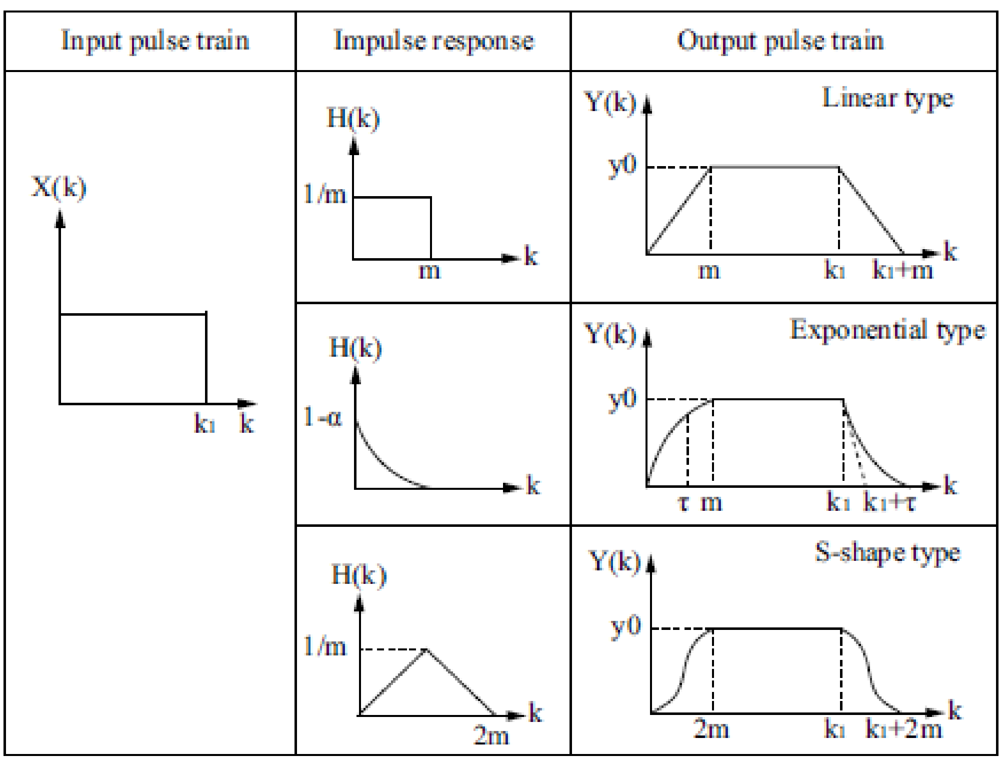

Fig. 4.4 Input and output pulse train profiles

Consequently, the Acc/Dec pulse profile generated by passing the input signal Vi through the above-mentioned filters can be represented by a recursive equation. **Equation 4.7, Eq. 4.8, and Eq. 4.9** are recursive equations for obtaining the linear type Acc/Dec pulse profile, the exponential-type Acc/Dec pulse profile, and the S shape-type Acc/Dec pulse profile, respectively,

$$V_{LO}(k) = \frac{1}{m}(V_i(k) - V_i(k-m)) + V_0(k-1) \qquad (4.7)$$

$$V_{EO}(k) = (1-\alpha)(V_i(k) - V_i(k-1)) + V_0(k-1) \qquad (4.8)$$

$$V_{SO}(k) = \frac{1}{m}(V_i(k) - V_i(k-m)) + V_{Otemp}(k-1) \qquad (4.9)$$

where $V_{Otemp}(k) = \frac{1}{m}(V_{Otemp}(k) - V_{Otemp}(k-m)) + V_O(k-1)$

Accordingly, the software Acc/Dec control algorithm is a relatively simple recursive equation and, therefore, has the merit of short calculation time.

2.2 Acc/Dec Control by Digital Circuit

Since the processing time of the Acc/Dec control method based on a digital circuit is very short, it has been used when the performance of CPUs was low. Hardware devices such as a shift register, a divider and an accumulator are used for implementing the Linear-type Acc/Dec control, the Exponential-type Acc/Dec control, and the S shape Acc/Dec control. However, as CPU performance has improved, the hardware type Acc/Dec control has been replaced by the software type Acc/Dec control that includes the same processing step as for the digital circuit.

In the ADCAI method, the pulse profile from rough interpolation is used as input of the Acc/Dec control circuit. The Acc/Dec control circuit plays the role of smoothing the change of pulse amount at the beginning and the end of a pulse profile.

In the following sections, three kinds of the Acc/Dec control algorithm will be addressed; **a Linear-type Acc/Dec control, an Exponential-type Acc/Dec control and an S-shaped Acc/Dec control.**

2.3 Acc/Dec Control Machining Errors

As mentioned above, because Acc/Dec control with ADCAI is applied separately for each axis, the path after Acc/Dec deviates from the programmed path.

In the case of a linear path on the XY plane, because the speed ratio between the Xand Y-axes before and after applying Acc/Dec control is constant, machining error due to Acc/Dec control does not occur. However, in the case of a circular path on the XY plane, the speed of the X- and Y-axes input to the Acc/Dec control circuit is actually a sine wave or cosine wave form. After passing through an Acc/Dec control filter (or circuit), the speed profiles of the X- and Y-axes are changed to a sine wave or cosine wave whose beginning and end are distorted.

In this section, the machining error for Linear Type Acc/Dec control, S-shape-type Acc/Dec control, and Exponential-type Acc/Dec control are discussed with respect to a circular path. For convenience of explanation, we assume that the feedrate for a circular path is F(mm/min) and the radius of the circular path is R(mm). Then the speed of each axis is given by Eq. 4.32.

$$\begin{aligned} V_x(t) &= -Rw\sin wt \\ V_y(t) &= Rw\cos(wt) \\ w &= \frac{F}{R} \end{aligned} \tag{4.32}$$

By applying Laplace transformation, Eq. 4.32 can be converted into Eq. 4.33.

$$\begin{aligned} V_x(s) &= -Rw\frac{w}{s^2+w^2} \\ V_y(s) &= Rw\frac{w}{s^2+w^2} \end{aligned} \tag{4.33}$$

By using Eq. 4.32 and Eq. 4.33, the machining error occurring with the three Acc/Dec control methods previously described are addressed during circular machining.

According to **Table 4.5** the machining error is proportional to the square of the feedrate and the Acc/Dec time. It is also in inverse proportion to the radius of the circular path. Therefore, from this, we know that the higher the feedrate the longer the Acc/Dec time, the shorter the radius of the circular path and the larger the machining error. We also know that the accuracy of the S-shape-type Acc/Dec control is better than that of the alternatives.

Table 4.5 Machining error due to Acc/Dec filter

Control type	Machining Error	Remarks
Linear	$\Delta R = \frac{F^2\tau^2}{24R}$	F: Feedrate
Exponential	$\Delta R = \frac{F^2\tau^2}{2R}$	τ: Time constant
S-shape	$\Delta R = \frac{F^2\tau^2}{48R}$	R: Radius of circle