Mathematics II

2017-2018

Class: 2^{nd} Year

Division: All divisions

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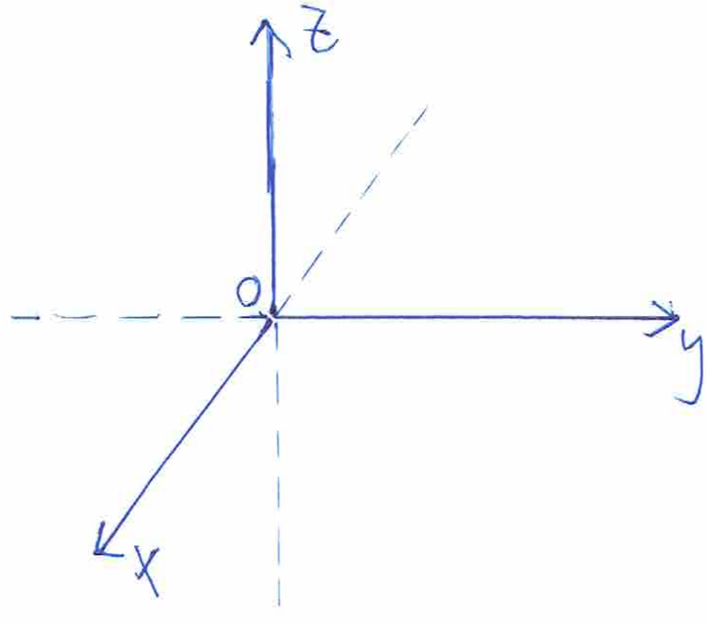
Vectors in 2- and 3-Dimensional Space

① First, we start with locating points in 3-dimensional space. We need three axes, the first two, the x-axis and y-axis we know them from our previous study. The third axis, the z-axis, is perpendicular to both of the x- and y-axis. They are sketched like:



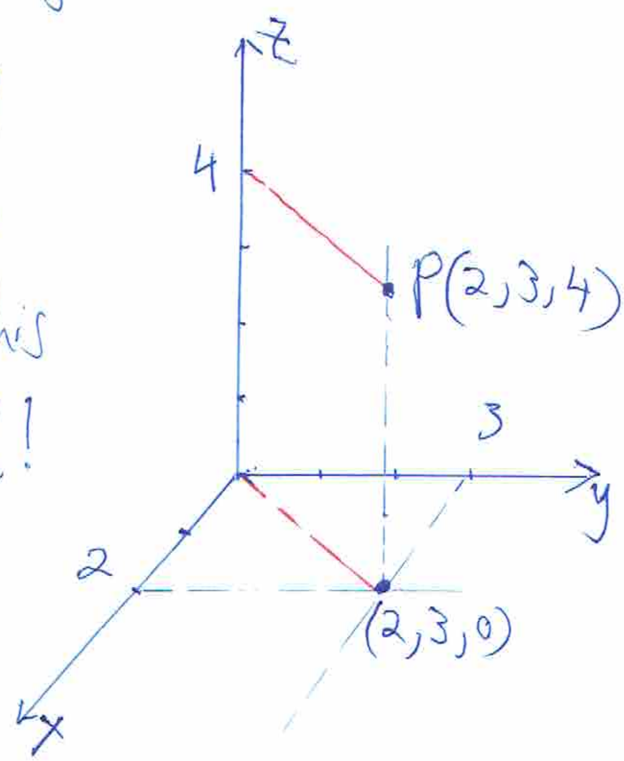
Here, the solid lines are the positive parts of the axes and the dashed lines are the negative. The origin $O=(0,0,0)$ is the intersection of all the axes. This 3-dim. space is denoted $\mathbb{R}^3$.

Now a point like $P=(2,3,4)$, for example, means: move 2 steps along x-axis,

3 steps along y-axis and

4 steps along z-axis.

Notice, that the dashed lines are here only to help us "imagine" that this is really 3-dimensional!



The blue dashed lines are parallel to the axes. The bottom ones intersect at $(2,3,0)$. The two red lines are also parallel.

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② The distance between two points $P_1(x_1,y_1,z_1)$ and $P_2(x_2,y_2,z_2)$ is given by:

$$\boxed{d(P_1,P_2)=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}$$

This follows from the Pythagoren theorem

The details are in the book. It follows that the distance between any point $P_1(x_1, y_1, z_1)$ and the origin $O(0,0,0)$ is

$$\sqrt{x_1^2 + y_1^2 + z_2}.$$

Notice that the set of all points having a distance r from the origin form a sphere $\sqrt{x^2+y^2+z^2} = r$ or $x^2+y^2+z^2 = r^2$. The radius is of course r. In general the equation of the sphere with radius and centered at, say, (h, k, l) is given by:

$$\boxed{(x-h)^2 + (y-k)^2 + (z-l)^2 = r^2}$$

Any quantity that has both magnitude and direction is called a *vector*. If it has only magnitude it is called a *scalar*.

A vector is represented by a directed line segment. The direction of the line segment is the direction of the vector and the length of the segment is the magnitude of the vector.

* / تذكر / ال vectors هو ليس قطعة مستقيمة موجهة وانما هو يمثل بـ قطعة مستقيمة موجهة تمثل وتشير الى اتجاه الفضاء ككل.

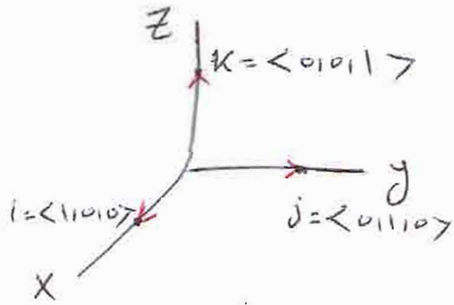
تذكر

* لاحظ :

$i \cdot i = 1$

$j \cdot j = 1$

$K \cdot K = 1$



يمثل متجهة يتحرك حركة واحدة باتجاه الـ المحور X وصفر البقية

where: i, j, K are the standard basis vectors

If $\vec{P} = \langle x, y, Z \rangle$ we can be write as:

$$= \langle x, 0, 0 \rangle + \langle 0, y, 0 \rangle + \langle 0, 0, Z \rangle$$

$$= x\langle 1, 0, 0 \rangle + y\langle 0, 1, 0 \rangle + Z\langle 0, 0, 1 \rangle$$

$$= xi + yj + ZK$$

وهذا يعني انه من الممكن ان نكتب المتجهة بهذه الطريقة ...

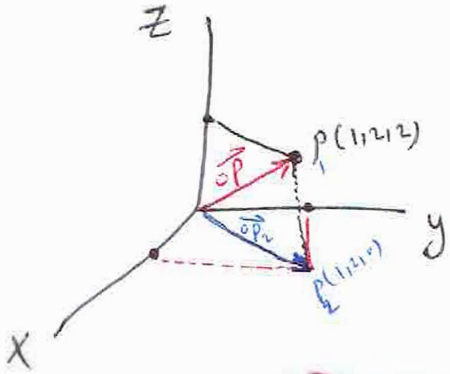
Ex/ $\vec{P} = \langle 1, 2, 2 \rangle$

$$\Rightarrow \vec{P} = i + 2j + 2K$$

لاحظ / العلاقة بين $P(1,2,2)$ وبين
$\vec{OP} = \langle 1,2,2 \rangle$

تمثل نقطة في الفضاء

يمثل متجهة لنقطة الانطلاق $(0,0,0)$



ويطلق على هذا المتجهة بـ position "standard" vector.

Ex (2) Find $\vec{a}\cdot\vec{a}$ where $\vec{a} = \langle 3,4,0 \rangle$

Sol:-

$\vec{a}\cdot\vec{a} = \langle 3,4,0 \rangle \cdot \langle 3,4,0 \rangle$
$= 25$

also

$\vec{a}\cdot\vec{a} = |\vec{a}||\vec{a}|\cos(0)$
$= (|\vec{a}|)^2 \cdot 1$
$= 25$

ح.ع.و

(3) Addition of vectors :

If $\vec{a} = \langle x_1, y_1, z_1 \rangle$ & $\vec{b} = \langle x_2, y_2, z_2 \rangle$

then " $\vec{a} + \vec{b} = \langle x_1 + x_2, y_1 + y_2, z_1 + z_2 \rangle$ "

بينما while the multiply avector $\vec{a} = \langle x, y, z \rangle$ in ascalar c

$\Rightarrow \vec{ca} = \langle cx, cy, cz \rangle$

if $c > 0 \Rightarrow \vec{a}$ & $\vec{ca}$ have the same direction.

but $c < 0 \Rightarrow \vec{a}$ & $\vec{ca}$ have " opposite direction.

أي انا بتشاهين متعاكسين

(4) Dot prodect :-

توجد طريقتين لحسابه هي

(1) $\vec{a}\cdot\vec{b} = \langle x_1, y_1, z_1 \rangle \cdot \langle x_2, y_2, z_2 \rangle$

$= x_1 x_2 + y_1 y_2 + z_1 z_2 \longrightarrow$ "and is the number " but is not vector

(2) $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta \longrightarrow$ يستخدم عند وجود angle $\Rightarrow$ " $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$ "

Ex (1) Find $i \cdot j$

Sol:- $i \cdot j = \langle 1,0,0 \rangle \cdot \langle 0,1,0 \rangle$
$= 0$

also

$i \cdot j = |\vec{i}||\vec{j}|\cos 90 = 0$

ح.ع.و

*/ **unit vector** : A vector $\vec{v}$ of the length 1 is called a unit vector.

U_v

*/ في حالة طلب ايجاد ال unit vector " "

$$\Rightarrow U_v = \frac{\vec{v}}{|\vec{v}|} \quad , \text{ where } \vec{v} = \langle v_1, v_2, v_3 \rangle .$$

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*/

def/ vectors $\vec{u}$ and $\vec{v}$ are orthogonal (perpendicular) iff $\vec{u} \cdot \vec{v} = 0$. (i.e $\theta = \frac{\pi}{2} = 90°$). $\cos(\frac{\pi}{2}) = 0$ (*)

Ex $u = \langle 3, -2 \rangle$ & $v = \langle 4, 6 \rangle$ are you orthogonal?

Sol:- $\vec{u} \cdot \vec{v} = 12 - 12 = 0$

$\therefore$ u, v are orthogonal.

*/ $\mathbf{0}$ is orthogonal to every vector $\vec{u}$.

"مهم جداً"

يعني / ان يكونا بنفس الاتجاه او يكونا في متعاكسين

*/ **def**/ vectors $\vec{u}$ & $\vec{v}$ are parallel (متوازيان) if the angle (الزاوية المحصورة بينهما) θ is Zero or π.

Theorem/ the vectors $\vec{u}$ & $\vec{v}$ are parallel $\Leftrightarrow$ $\vec{u} = c\vec{v}$, where c is scalar.

Ex/ prove that $\vec{V} = i + 3j - 2K$ & $\vec{u} = -3i - 9j + 6K$ are parallel ?

proof/ method «1»

$$\vec{u} \cdot \vec{V} = (1)(-3) + (3)(-9) + (-2)(6)$$

$$= -3 - 27 - 12 = -42$$

$$|\vec{u}| = \sqrt{(-3)^2 + (-9)^2 + (6)^2} = \sqrt{126}$$

$$|\vec{V}| = \sqrt{(1)^2 + (3)^2 + (-2)^2} = \sqrt{14}$$

قانون ← $\because \cos\theta = \dfrac{\vec{u} \cdot \vec{V}}{|\vec{u}||\vec{V}|} = \dfrac{-42}{\sqrt{14} \cdot \sqrt{126}} = -1$

$$\therefore \theta = \pi \Rightarrow \vec{u} \parallel \vec{V}$$

و.هـ.م

method «2»

نعمل على اي واحد منهم وبالاخص الذي يمكن ان نبسط مفرداته باستخراج العامل المشترك .

لذلك سنعمل على $\vec{u}$

$$\vec{u} = -3i - 9j + 6K = -3(i + 3j - 2K)$$

$$= -3\vec{V}$$

i.e $\vec{u} = c\vec{V}$

then $\vec{u} \parallel \vec{V}$

و.هـ.م

باستخدام ال cross product

method «3»

مهم If u & V be a nonzero vectors then $u \parallel V$ «are parallel» $\Longleftrightarrow u \times V = 0$