solution :-

a) $\lim\limits_{x\to -2^-} f(x) = \lim\limits_{x\to -2} \dfrac{1}{x+2} = -\infty$

$\lim\limits_{x\to -2^+} f(x) = \lim\limits_{x\to -2^+} (x^2-5) = -1$

Thus $\lim\limits_{x\to -2^-} f(x) \neq \lim\limits_{x\to -2^+} f(x)$

$\therefore \lim\limits_{x\to -2} f(x)$ does not exist.

b) $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} (x^2-5) = 0^2-5 = -5$

c) $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} (x^2-5) = 3^2-5 = 4$

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} \sqrt{x+13} = \sqrt{3+13} = 4$

since the one-sided limits are equal, we have

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^-} f(x) = 4$

Thus $\lim\limits_{x\to 3} f(x) = 4$

Theorem:- Let a and k be real numbers.

$\lim\limits_{x\to a} k = k$; $\lim\limits_{x\to a} x = a$, $\lim\limits_{x\to 0^-} \dfrac{1}{x} = -\infty$, $\lim\limits_{x\to 0^+} \dfrac{1}{x} = +\infty$

Note: (Sometimes, one or both of one-sided limits may fail to exist which implies that two-sided limit does not exist)

i.e. if $\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x)$,

then the $\lim_{x \to a} f(x)$ does not exist.

Examples:— Find:

1) $\lim_{x \to 2} (2x^3 - 6x + 4) = \lim_{x \to 2} 2x^3 - \lim_{x \to 2} 6x + \lim_{x \to 2} 4$

$= 2\lim_{x \to 2} x^3 - 6\lim_{x \to 2} x + \lim_{x \to 2} 4$

$= 2(8) - 6(2) + 4$

$= 16 - 12 + 4$

$= 8$

2) $\lim_{x \to -5} \left(\frac{3x+1}{5}\right)$

$= \frac{3\lim_{x \to -5} x + \lim_{x \to -5} (1)}{\lim_{x \to -5} (5)} = \frac{3(-5)+1}{5}$

$= \frac{-14}{5}$

3) Let

$$f(x) = \begin{cases} \frac{1}{(x+2)} & , \ x < -2 \\ x^2 - 5 & , \ -2 < x \leq 3 \\ \sqrt{x+13} & , \ x > 3 \end{cases}$$

Find a) $\lim_{x \to -2} f(x)$ b) $\lim_{x \to 0} f(x)$ c) $\lim_{x \to 3} f(x)$

Limits النهايات

Def: By $\lim_{x \to a} f(x) = L$, L is a real num
we mean the limit of $f(x)$ as x approache
a is L.

Def: ((One-sides limits))

① $\lim_{x \to a^+} f(x) = L$ which is read:
the limit of $f(x)$ as x approaches a from the right is L.

② $\lim_{x \to a^-} f(x) = L$ which is read:
the limit of $f(x)$ as x approaches a from the left is L.

Def: ((The Relationship between one-sided and two sided Limits))

The two-sided limit of a function $f(x)$ exists at a if and only if both of the one-sided limits exist at a and have the same value that is;

$$\lim_{x \to a} f(x) = L \quad \text{if and only if} \quad \lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$$

" Hyperbolic functions " دوال الزائدية

Definition :—

$$\sinh x = \frac{e^x - e^{-x}}{2}$$ « Hyperbolic sine »

$$\cosh x = \frac{e^x + e^{-x}}{2}$$ « Hyperbolic cosine »

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$ « Hyperbolic tangent »

$$\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$ « Hyperbolic cotangent »

$$\operatorname{sech} x = \frac{2}{e^x + e^{-x}} = \frac{1}{\cosh x}$$ « Hyperbolic secant »

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$ « Hyperbolic cosecant »

Remark ;—

$$\sinh 0 = \frac{e^0 - e^{-0}}{2} = \frac{1-1}{2} = \frac{0}{2} = 0$$

$$\cosh 0 = \frac{e^0 + e^{-0}}{2} = \frac{1+1}{2} = 1$$

$$\sinh 2 = \frac{e^2 - e^{-2}}{2} \approx 3.6269$$

Theorem :-

$$\cosh^2 x - \sinh^2 x = 1$$

proof :-

$$L.H.S. \Rightarrow \cosh^2 x - \sinh^2 x$$

$$\Rightarrow \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$= \frac{1}{4}\left(e^{2x} + 2e^0 + e^{-2x}\right) - \frac{1}{4}\left(e^{2x} - 2e^0 + e^{-2x}\right)$$

$$= 1$$

$$e^x \cdot e^x = e^{2x}$$

$$e^x \cdot e^{-x} = e^0 = 1$$

$$= R.H.S.$$

Theorem :-

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\cosh^2 x - 1 = \operatorname{csch}^2 x$$

$$\cosh(-x) = \cosh x$$

$$\sinh(-x) = -\sinh x$$

$$\sinh 2x = 2\sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$\cosh 2x = 2\sinh^2 x + 1$$

Ex: Find a value for the constant k that make

$$f(x) = \begin{cases} \dfrac{\sin 3x}{x}, & x < 0 \\ k, & x = 0 \end{cases}$$

continuous at $x = 0$.

« The Derivative » مشتقة

« Differentiation » التفاضل

(Sheet no. 2) راجع

نظرية

Theorem:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1 \quad , \quad \lim_{x \to 0} \frac{1-\cos x}{x} = 0$$

$$\lim_{x \to 0} \frac{\tan x}{x} = 1$$

proof: $\left\{ \lim_{x \to 0} \frac{1-\cos x}{x} = 0 \right\}$ بالتعويض المباشر $\rightarrow \frac{0}{0}$

$$\Rightarrow \lim_{x \to 0} \left[\frac{1-\cos x}{x} \cdot \frac{1+\cos x}{1+\cos x} \right]$$

$$= \lim_{x \to 0} \frac{\sin^2 x}{x\,(1+\cos(x))}$$

$$= \left(\lim_{x \to 0} \frac{\sin x}{x}\right) \left(\lim_{x \to 0} \frac{\sin x}{1+\cos x}\right)$$

$$= (1)(0)$$

$$= 0$$

Examples: Find :-

1) $\lim_{x \to 0} \frac{\tan x}{x}$

$$= \lim_{x \to 0} \left(\frac{\sin x}{x} \cdot \frac{1}{\cos x} \right) = (1)(1) = 1$$

2) $\lim_{\theta \to 0} \frac{\sin 2\theta}{\theta}$ if $\theta \to 0$ then $2\theta \to 0$

$$\Rightarrow \lim_{\theta \to 0} \left(\frac{2}{2} \frac{\sin 2\theta}{\theta} \right)$$

$$\Rightarrow 2 \left(\lim_{\theta \to 0} \frac{\sin 2\theta}{2\theta} \right) = 2(1) = 2$$

examples:- Find

1) $\lim_{x \to 2} \frac{x^2 - 4}{x - 2} \rightarrow \frac{0}{0}$ بالتعويض المباشر

نستخدم التحليل الى عوامل اولية

$$= \lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)}$$

$$= 4$$

2) $\lim_{x \to 0} \frac{x}{\sqrt{x+1} - 1} \rightarrow \frac{0}{0}$ بالتعويض المباشر

نستخدم الضرب بالمرافق

$$\lim_{x \to 0} \frac{x}{\sqrt{x+1} - 1} \cdot \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1}$$

$$= \lim_{x \to 0} \frac{x(\sqrt{x+1} + 1)}{(x+1) - 1}$$

$$= \lim_{x \to 0} \frac{x(\sqrt{x+1} + 1)}{x}$$

$$= 2$$

المرحلة الاولى
قسم هندسة الانتاج والمعادن
[illegible]
2015

رياضيات I
فرع ... هندسة التصميم والتصنيع بمساعدة الحاسوب

The equation of straight and circle :-

معادلة الخط المستقيم والدائرة

An equation for a line is an equation that is satisfied by the coordinates of the points that lie on the line and is not satisfied by the coordinates of the points that lie elsewhere.

معادلة الخط المستقيم هي :-

[illegible] x و y [illegible]

[illegible]

2) [illegible]

[illegible] $x=a$ [illegible]

[illegible] x [illegible] $x=a$ [illegible] y

[illegible] $y=a$ [illegible]

[illegible] $y=a$ [illegible]

Defs: ① The standard equation for the vertical line (عمودي) through the point (a,b) is $x=a$.

② The standard equation for the horizontal line (افقي) through the point (a,b) is $y=b$.

③ The point – slope equation of the line through the point (x_1, y_1) with slope m is

معادلة ميل المقطع للخط المار بالنقطة (x_1, y_1).

$$y - y_1 = m(x - x_1)$$

معادلة الخط المستقيم المار بنقطتين

The equation for the line through the two points

ii) (−2, 0) , (−2, −2) → ∵ $x_1 = x_2 = -2$ ⇒ $x = -2$ is the equation for the line

iii) (1, 2) , (3, 1) :

Sol.:- the equation for the line through the two points is:

$$y - y_1 = m(x - x_1)$$

$$\frac{y - y_1}{x - x_1} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y - y_1}{x - x_1} = \frac{1 - 2}{3 - 1}$$

$$\frac{y - y_1}{x - x_1} = \frac{-1}{2}$$

$$\therefore 2(y - 2) = -(x - 1)$$

$$2y + x = 4 + 1$$

$$\therefore 2y + x - 5 = 0$$

(Equations for lines)

$x = a$ Vertical line through (a, b)

$y = b$ Horizontal line through (a, b)

$y = mx + b$ Slope - intercept equation

ii) (-2, 0) , (-2, -2) ⟹ ∵ $x_1 = x_2 = -2$ ⟹ $x = -2$ is the equation for the line

iii) (1, 2) , (3, 1)

Sol.:- the equation for the line through the two points is:

$$y - y_1 = m(x - x_1)$$

$$\frac{y - y_1}{x - x_1} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y - y_1}{x - x_1} = \frac{1 - 2}{3 - 1}$$

$$\frac{y - y_1}{x - x_1} = \frac{-1}{2}$$

$$\therefore 2(y - 2) = -(x - 1)$$

$$2y + x = 4 + 1$$

$$\therefore 2y + x - 5 = 0$$

(Equations for lines)

$x = a$ Vertical line through (a, b)

$y = b$ Horizontal line through (a, b)

$y = mx + b$ Slope - intercept equation

$y - y_1 = m(x - x_1)$ point - slope equation

$Ax + By = C$ } A, B, C (a, b, c) constant
$ax + by = c$ | A, B (a, b) are not both zero

(General liner equation)

Exc: Write the equation for the line with the given slope:-

i) $m = 3$, $b = -2$

ii) $m = -1$, $b = 2$

iii) $m = \frac{1}{3}$, $b = -1$

Sol. $y = mx + b$ slope — intercept equation
معادلة مقطع الميل

i) $y = 3x - 2$

ii) $y = -x + 2$

iii) $y = \frac{1}{3}x - 1$

Circles:

Def: A circle is the set of points in a plane whose distance from a given fixed point in the plane is a constant

Equations for Circles: the standard equation for the circle of radius a centered at the point (h, k) is:- $(x-h)^2 + (y-k)^2 = a^2$

$m = \frac{4-(-1)}{3-(-2)} = \frac{5}{5} = 1$

Now we have the point - slope equation

$y - y_1 = m(x - x_1)$, $(x_1, y_1) = (-2, -1)$

$\therefore y + 1 = 1(x + 2)$

$\therefore y = x + 1$

Note: If we put $(x_1, y_1) = (3, 4)$ we have the same result

since $y - 4 = 1(x - 3)$

$y - 4 = x - 3$

$y = x + 1$

Note: If $x_1 = x_2$ then the equation for the line is $x = x_1$

EXC: 1) Write the equation for

i) the vertical line ii) the horizontal line

through the given point.

$(2,3) \rightarrow$ i) $x = 2$, ii) $y = 3$

$(-4,0) \rightarrow$ i) $x = -4$, ii) $y = 0$

$(0,b) \rightarrow$ i) $x = 0$, ii) $y = b$

2) Find an equation for the line through the two points

i) $(1,1)$ $(1,2)$

sol: $\because x_1 = x_2 = 1 \Rightarrow x = 1$ is the equation for the line

ii) $(-2,0)$, $(-2,-2)$ $\Rightarrow$ $\because x_1 = x_2 = -2$ $\Rightarrow$ $x = -2$ is the equation for the line

iii) $(1,2)$, $(3,1)$

Sol.:- the equation for the line through the two points is:

$$y - y_1 = m(x - x_1)$$

$$\frac{y - y_1}{x - x_1} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{y - y_1}{x - x_1} = \frac{1-2}{3-1}$$

$$\frac{y - y_1}{x - x_1} = \frac{-1}{2}$$

$$\therefore 2(y-2) = -(x-1)$$

$$2y + x = 4 + 1$$

$$\therefore 2y + x - 5 = 0$$

(Equations for lines)

$x = a$ Vertical line through (a,b)

$y = b$ Horizontal line through (a,b)

$y = mx + b$ Slope - intercept equation

$m = \frac{4-(-1)}{3-(-2)} = \frac{5}{5} = 1$

Now we have the point-slope equation

$y - y_1 = m(x - x_1)$, $(x_1, y_1) = (-2, -1)$

$\therefore y + 1 = 1(x + 2)$

$\therefore y = x + 1$

Note: If we put $(x_1, y_1) = (3, 4)$ we have the same result

since $y - 4 = 1(x - 3)$

$y - 4 = x - 3$

$y = x + 1$

Note: If $x_1 = x_2$ then the equation for the line is $x = x_1$

Exc: 1) Write the equation for

i) the vertical line ii) the horizontal line through the given point.

$(2,3) \rightarrow$ i) $x = 2$, ii) $y = 3$

$(-4,0) \rightarrow$ i) $x = -4$, ii) $y = 0$

$(0,b) \rightarrow$ i) $x = 0$, ii) $y = b$

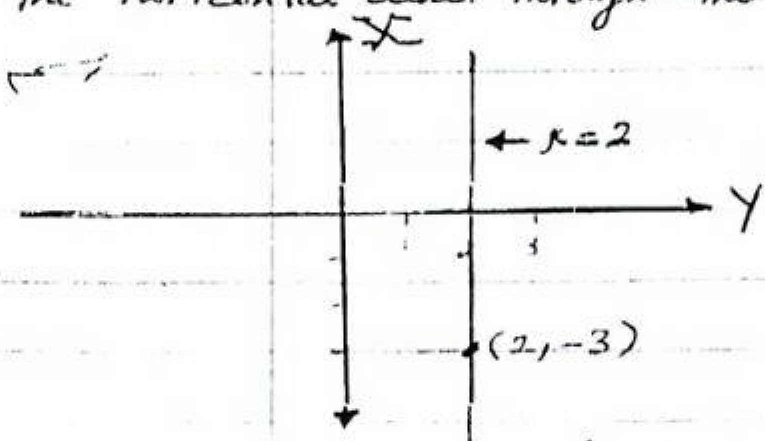
2) Find an equation for the line through the two points

i) $(1,1)$ $(1,2)$

Sol: $\because x_1 = x_2 = 1 \Rightarrow x = 1$ is the equation for the line examples:

1) The standard equation of the vertical line through point $(2,-3)$ is $x = 2$

2) The equation of the horizontal line through the point $(1,2)$ is $y = 2$ (Ex2)



example :- Write an equation for the line through the point $(1,2)$ with slope $m = -3/4$.

Sol. : we have $x_1 = 1$, $y_1 = 2$, $m = -3/4$

$\therefore \; y - y_1 = m(x - x_1)$ معادلة خط لنقطة المستقيم

$y - 2 = -\frac{3}{4}(x - 1)$

$\therefore \; y = -\frac{3}{4}x + \frac{11}{4}$

Note: The slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $\quad x_1 \neq x_2$

where (x_1, y_1), (x_2, y_2) are two points on the line L. نقطتين معلومتين

example: Write the equation for the line through $(-2,-1)$ and $(3,4)$

Sol. : The slope $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

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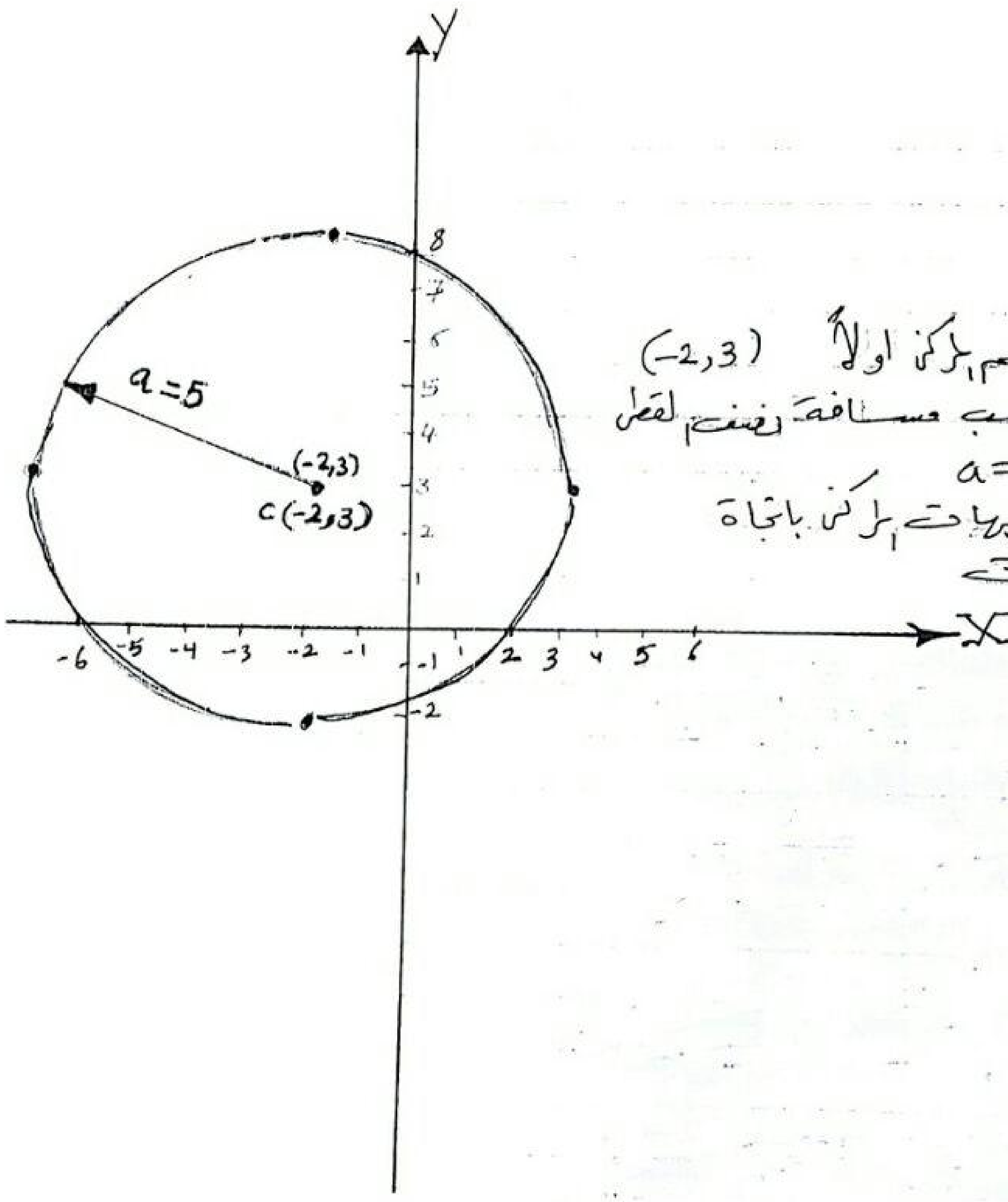


نرسم المركز اولاً (-2,3)

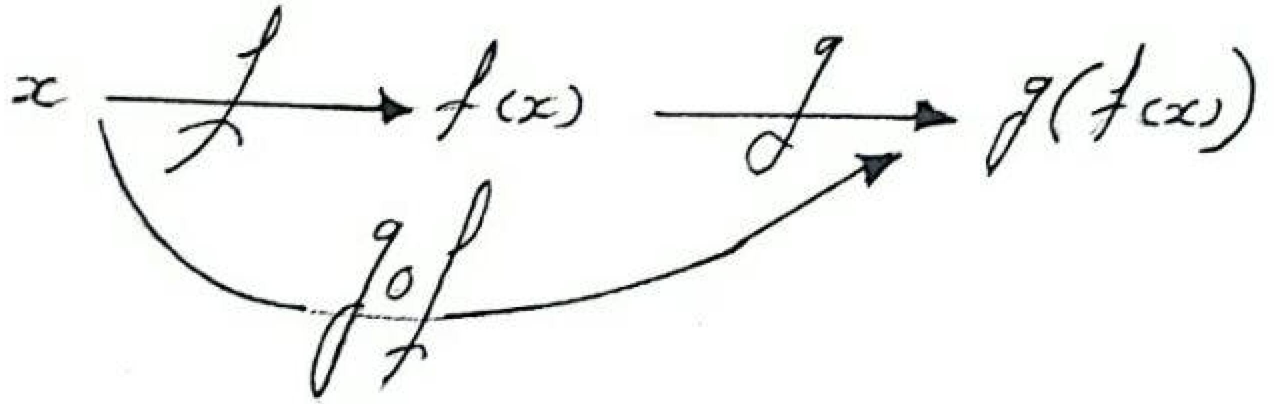
ثم نحسب مسافة نصف القطر

$a=r=5$

من جميع جهات المركز باتجاه الاحداثيات

Composition of functions:

two functions f and g can be composed when the range of the first function lies in the domain of the second.



The composite $g \circ f$ which is read as " g of f "
Thus the value of $g \circ f$ at x is :—

$$(g \circ f)(x) = g(f(x))$$

example: Find a formula for $g(f(x))$ if :—

1) $f(x) = x^2$ and $g(x) = x - 7$

$$(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 7$$

to find the value of $g(f(2)) = 2^2 - 7 = 4 - 7 = -3$

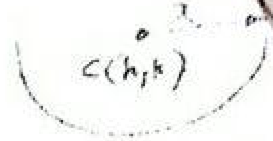
2) $f(x) = \sin x$ and $g(x) = \frac{-x}{2}$

$$(g \circ f)(x) = g(f(x)) = g(\sin x) = -\frac{\sin x}{2}$$

3) $f(x) = x^2$ and $g(x) = x - 7$ to compute $(f \circ g)(x)$

Example: if the center at the origin and with r the equation for the circle is:

$$h = k = 0 \Rightarrow x^2 + y^2 = a^2$$



Example: Find the circle through the origin with center at $c(2,-1)$

Sol.

with $(h,k) = (2,-1)$ so

$(x-h)^2 + (y-k)^2 = a^2$ take the form

$(x-2)^2 + (y+1)^2 = a^2$

since the circle goes through the origin, $x = y = 0$ must satisfy the equation.

Hence $(0-2)^2 + (0+1)^2 = a^2 \Rightarrow a^2 = 5$

$\therefore$ The equation $(x-2)^2 + (y+1)^2 = 5$

Example: Find the center and radius of the circle

$$x^2 + y^2 + 4x - 6y = 12 ,$$

then draw the circle

Sol. We complete the squares in the x terms and y terms and get

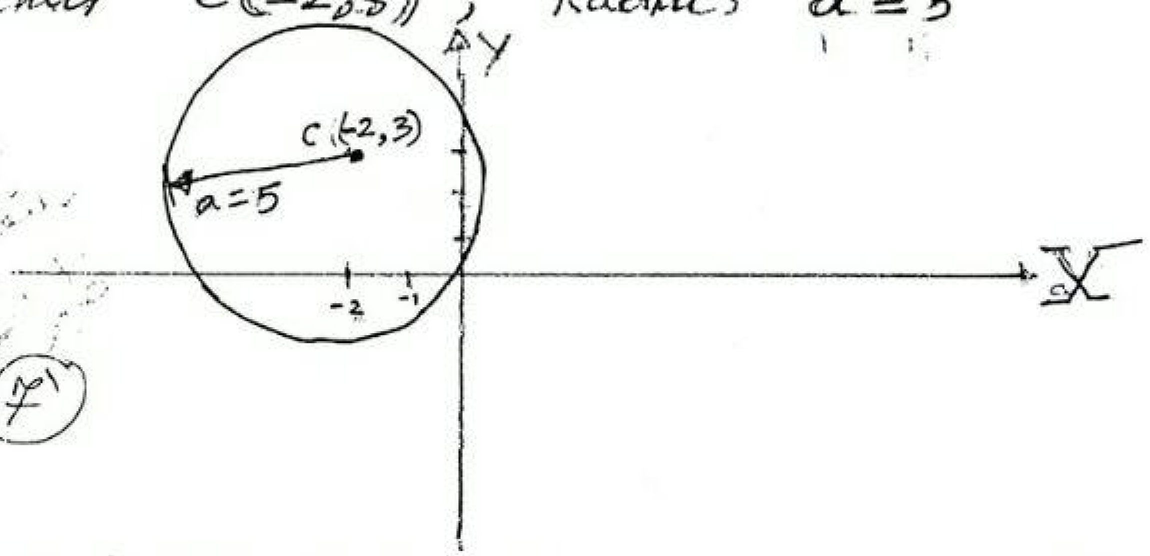
$$(x^2+4x+4)+(y^2-6y+9)=12+4+9$$

$$\Rightarrow (x+2)^2+(y-3)^2=25$$

$$(x-h)^2+(y-k)^2=a^2$$

$$\Rightarrow (h,k)=(-2,3) \quad \text{and} \quad a^2=25$$

$\Rightarrow$ Center $c(-2,3)$, Radius $a=5$



(7')

Exc 1) Find the circle with center $c(h,k)$ and radius a

a) $c(0,2)$, $a=2$

$$\Rightarrow (h,k)=(0,2)$$
$$\Rightarrow (x-0)^2+(y-2)^2=4$$
$$x^2+y^2-4y+4-4=0$$
$$x^2+y^2-4y=0$$

b) $c(3,-4)$, $a=5$

$$\Rightarrow (h,k)=(3,-4)$$
$$\Rightarrow (x-3)^2-(y+4)^2=25$$
$$\Rightarrow x^2-6x+9+y^2+8y+16=25$$
$$\Rightarrow x^2-6x+y^2+8y=0$$
$$\Rightarrow x^2+y^2-6x+8y=0$$

5) $f(x) = \sin x \quad , x \in \mathbb{R}$

$f(-x) = \sin(-x) = -\sin x = -f(x)$

$\therefore$ f is odd function

6) $f(x) = x^2+1 \quad , x \in \mathbb{R}$ (EXC)

$f(-x) = (-x)^2+1 = x^2+1 = f(x)$

$\therefore$ f is even function

7) $f(x) = \frac{1}{x^2-1} \quad , x \in \mathbb{R}$

$f(-x) = \frac{1}{(-x)^2-1} = \frac{1}{x^2-1} = f(x)$

$\therefore$ f is even function

8) $f(x) = \frac{x}{x^2-1}$ (EXC)

$f(-x) = \frac{-x}{(-x)^2-1} = -\frac{x}{x^2-1} = -f(x)$

$\therefore$ f is odd function

9) $f(x) = x+1$

$f(-x) = -x+1$

$\neq f(x)$ or $-f(x)$

$\therefore$ f neither even or odd function

Even and odd functions:

Def:– A function f is said to be (even) if $f(-x) = f(x)$ (even) and it is said to be (odd) if $f(-x) = -f(x)$ (odd), for all $x \in D_f$

examples:– Is the following function are even or odd:–

1) $f(x) = x^2$, $x \in \mathbb{R}$

$f(-x) = (-x)^2 = x^2 = f(x)$

$\therefore f$ is (even) function

2) $f(x) = \cos x$ $\quad -2\pi < x < 2\pi$

$f(-x) = \cos(-x) = \cos x = f(x)$

$\therefore f$ is (even) function

3) $f(x) = 5$, $x \in \mathbb{R}$

$f(-x) = 5 = f(x)$

$\therefore f$ is even function

4) $f(x) = x^3$, $x \in \mathbb{R}$

$f(-x) = (-x)^3 = -x^3 = -f(x)$

$\therefore f$ is odd function

$f(g(x)) = f(x-7) = (x-7)^2$

To find $f(g(2)) = (2-7)^2 = 25$

Now from 1) we have $(g \circ f)(2) = -3$

$(f \circ g)(2) = 25$

So $(f \circ g)(2) \neq (g \circ f)(2)$

Note: In general the composition function is not commutative

i.e. $f \circ g \neq g \circ f$

2) $f(x) = x+5$ and $g(x) = x^2 - 3$

$(g \circ f)(x) = g(f(x)) = g(x+5) = (x+5)^2 - 3$

$= x^2 + 10x + 25 - 3$

$\therefore (g \circ f)(x) = g(f(x)) = x^2 + 10x + 22$

to find $(g \circ f)(0) = 0^2 + 10(0) + 22 = 22$

Now $(f \circ g)(x) = f(g(x)) = f(x^2-3) = (x^2-3) + 5$

$\therefore f(g(x)) = x^2 + 2$

Now $(f \circ g)(0) = 0^2 + 2 = 2$

$\therefore (f \circ g)(0) \neq (g \circ f)(0)$

سما هوزاري

(54)

$$e^y = \frac{2x \mp \sqrt{4x^2+4}}{2}$$

$$\therefore e^y = x \mp \sqrt{x^2+1}$$

$$\Rightarrow e^y = x + \sqrt{x^2+1} \quad \text{((since } e^y > 0 \text{))}$$

$$\Rightarrow \ln e^y = \ln(x + \sqrt{x^2+1}) \quad \text{((taking natural logarithm))}$$

$$\Rightarrow y = \ln(x + \sqrt{x^2+1})$$

$$\Rightarrow \sinh^{-1} x = \ln(x + \sqrt{x^2+1})$$

ExC :— Find dy/dx :—

1) $y = \sqrt{4x + \cosh^2 5x}$

2) $y = \ln(\cosh^{-1} 2x)$

3) $y = \cosh^{-1}(\sin 3x)$

4) $y = e^x \operatorname{sech}^{-1} \sqrt{x}$

5) $y = \ln(\tanh e^{3x})$

6) $y = \dfrac{\sin x \sec x}{1 + x \tan 5x}$

7) $y = \dfrac{(x^2+1) \cot e^{2x}}{3 - \cos x \csc x}$

8) $f(x) = \sin^2 5x + \cos^2 3x$

9) $y = \tan^{-1}(\sinh x) + \tan(\sin^{-1} x)$

10) $y = \sqrt{x} \tan^3 \sqrt{x}$

11) $y = \tanh e^x + \sec^{-1}(\operatorname{csch} 2x)$

12) $y = [1 + \sin^3(x^5)]^{3/2}$

(53)

$\cosh x + \sinh x = e^{x}$

$\cosh x - \sinh x = e^{-x}$

$\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$

$\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$

$\sinh(x-y) = \sinh x \cosh y - \cosh x \sinh y$

$\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

Theorem

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$$

Proof:-

$\because y = \sinh^{-1} x \Rightarrow x = \sinh y$

$\therefore x = \sinh y = \dfrac{e^{y} - e^{-y}}{2}$

$\Rightarrow e^{y} - 2x - e^{-y} = 0$ « بحاصل ضرب طرفين في وسطين »

$\Rightarrow e^{2y} - 2x\,e^{y} - 1 = 0$ ((Multiply this equation by e^{y}))

نستخدم قانون المستقيم في تحليل لحل المعادلة

we have from above

① $(e^{y})^2 - 2x\, e^{y} - 1 = 0$

a (coefficient 1), $b = -2x$, $c = -1$

" Hyperbolic functions " دوال الزائدية

Definition :-

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$ « Hyperbolic sine »

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$ « Hyperbolic cosine »

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$ « Hyperbolic tangent »

$$\coth x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$ « Hyperbolic cotangent »

$$\operatorname{sech} x = \frac{2}{e^{x} + e^{-x}} = \frac{1}{\cosh x}$$ « Hyperbolic secant »

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^{x} - e^{-x}}$$ « Hyperbolic cosecant »

Remark :-

$$\sinh 0 = \frac{e^{0} - e^{-0}}{2} = \frac{1-1}{2} = \frac{0}{2} = 0$$

$$\cosh 0 = \frac{e^{0} + e^{-0}}{2} = \frac{1+1}{2} = 1$$

$$\sinh 2 = \frac{e^{2} - e^{-2}}{2} \approx 3.6269$$

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د. ... ضمير

Theorem :-

$$\cosh^2 x - \sinh^2 x = 1$$

Proof :-

$$L.H.S. \Rightarrow \cosh^2 x - \sinh^2 x$$

$$\Rightarrow \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$= \frac{1}{4}\left(e^{2x} + 2e^0 + e^{-2x}\right) - \frac{1}{4}\left(e^{2x} - 2e^0 + e^{-2x}\right)$$

$$= 1$$

$$e^x \cdot e^x = e^{2x}$$
$$e^x \cdot e^{-x} = e^0 = 1$$

$$= R.H.S.$$

Theorem :-

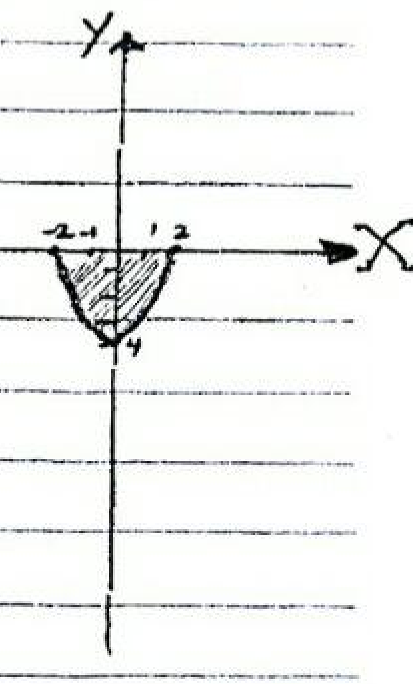
$$1 - \tanh^2 x = \operatorname{sech}^2 x$$
$$\cosh^2 x - 1 = \operatorname{csch}^2 x$$
$$\cosh(-x) = \cosh x$$
$$\sinh(-x) = -\sinh x$$
$$\sinh 2x = 2\sinh x \cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$\cosh 2x = 2\sinh^2 x + 1$$

The Area between the Curve $f(x)$ and x-axis :-

Note :- the area has positive value always.

- كثيراً عندما بعض الأسئلة نجد ان المساحة تكون بالسالبة والاشارة السالبة تدل على ان المساحة تحت محور السينات.
- اذا لم تعطى حدود التكامل في السؤال فنجدها عن طريق نقطة تقاطع منحني $f(x)$ مع محور السينات.

example :- Find the area between the x-axis and the curve $y = x^2 - 4$, for $-2 \le x \le 2$



$$A = \int_{-2}^{2} (x^2 - 4)\,dx$$

$$= \left(\frac{x^3}{3} - 4x\right)\Big]_{-2}^{2}$$

$$= \left(\frac{8}{3} - 8\right) - \left(\frac{-8}{3} + 8\right)$$

$$= \frac{8}{3} - 8 + \frac{8}{3} - 8$$

$$= \frac{16}{3} - 16$$

$$= \frac{16 - 48}{3}$$

$$\therefore A = -\frac{32}{3}$$

$$\therefore A = \frac{-32}{3}$$ ← اشارة السالبة تدل ان المنحني تحت محور السينات

$$\Rightarrow A = \left|\frac{-32}{3}\right| = \frac{32}{3} \text{ square unit}$$

Integration Methodes :

1) Integration by Substitution "التكامل بالتعويض"

2) Integration by parts. "التكامل بالتجزأة"

3) Integrating powers of Sine and cosine, Secant and tangent — "تكاملات دوال مثلثية" مرفوعة لأس

4) Rational functions and partial fractions. (تكاملات الكسور الجزئية) (التحليل إلى عوامل خطية و المسائل الجزئية)

5) Trigonometric substitutions ($\sqrt{a^2-x^2}$, $\sqrt{a^2+x^2}$, $\sqrt{x^2-a^2}$)

6) Integration by completing the square — Integrals involving ax^2+bx+c — تكاملات بواسطة إكمال المربع

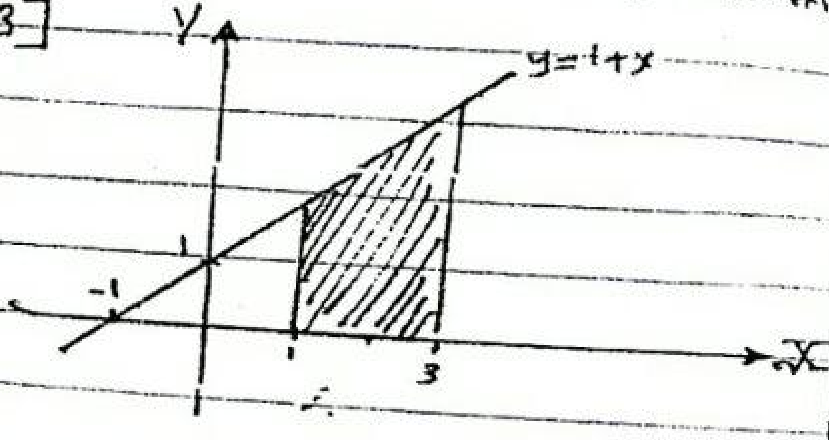
7) Numerical Integration — التكاملات العددية

- Simpson's Rule — قانون سمبسون
- Trapezoidal approximation — تقريب شبه المنحرف

Examples:

1) Find the area between the curve and x-axis

$f(x) = 1 + x \quad , \quad x \in [1,3]$



Sol.:

$A = \int_1^3 (1+x)\,dx$

$= x\Big|_1^3 + \frac{x^2}{2}\Big|_1^3$

$= (3-1) + \left(\frac{9}{2} - \frac{1}{2}\right)$

$= 2 + \left(\frac{8}{2}\right)$

$= 6$ (square unit)

2) $f(x) = 2x - x^2$

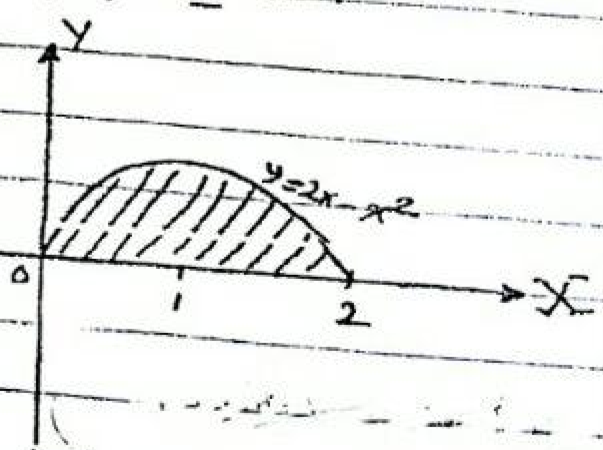
Sol. في هذه الحالة لا تعطى حدود التكامل وهي عبارة عن نقاط تقاطع المنحني $f(x)$ مع محور x.

Now we put $f(x) = y = 0$

then $f(x) = 2x - x^2 = 0$

$\Rightarrow x(2-x) = 0$

$\Rightarrow x = 0, 2$ « حدود التكامل »



Now $A = \int_0^2 (2x - x^2)\,dx$

$= \frac{2x^2}{2}\Big|_0^2 - \frac{x^3}{3}\Big|_0^2$

$= x^2\Big|_0^2 - \frac{x^3}{3}\Big|_0^2$

$= 4 - \frac{8}{3} = \frac{4}{3}$ (square unit)

1) Integration by substitution

التكامل بالتعويض

نلجأ الى التكامل بالتعويض عادة عندما يكون لدينا تكامل حاصل ضرب دالتين
(أو اقترانين) بحيث يكون مشتقة احدهما تساوي الآخر أو
تساوي الآخر مضروب بعدد ثابت.

Examples → Evaluate:

1) $\int \sec^2 x \tan x \, dx$

Let $u = \tan x \rightarrow du = \sec^2 x \, dx$

$\therefore \int \sec^2 x \tan x \, dx = \int u \, du$

$$= \frac{u^2}{2} + c$$

$$= \frac{(\tan x)^2}{2} + c$$

2) $\int 3x^2 (x^3+1)^4 \, dx$

Let $u = x^3 + 1 \rightarrow du = 3x^2 \, dx$

$\therefore \int 3x^2 (x^3+1)^4 \, dx = \int u^4 \, du$

$$= \frac{u^5}{5} + c$$

$$= \frac{(x^3+1)^5}{5} + c$$