Lecture

Mechanical Properties and Testing

The main goal of this chapter is to introduce the basic concepts associated with mechanical properties.

We will learn terms such as hardness, stress, strain, elastic and plastic deformation, viscoelasticity, and strain rate.

We will also review some of the testing procedures that engineers use to evaluate many of these properties.

These concepts will be discussed using illustrations from real-world applications.

The Tensile Test: Use of the Stress–Strain Diagram

The tensile test is popular since the properties obtained can be applied to design different components. The tensile test measures the resistance of a material to a static or slowly applied force. The strain rates in a tensile test are typically small (10^{-4} to 10^{-2} s^{-1}). A test setup is shown in Figure 6-5; a typical specimen has a diameter of 0.505 in. and a gage length of 2 in. The specimen is placed in the testing machine and a force F, called the **load**, is applied. A universal testing machine on which tensile and compressive tests can be performed often is used. A **strain gage** or **extensometer** is used to measure the amount that the specimen stretches between the gage marks when the force is applied. Thus, the change in length of the specimen (Δl) is measured with respect to the original length (l_0). Information concerning the strength, Young's modulus, and ductility of a material can be obtained from such a tensile test. Typically, a tensile test is conducted on metals, alloys, and plastics. Tensile tests can be used for ceramics; however, these are not very popular because the sample may fracture while it is being aligned. The following discussion mainly applies to tensile testing of metals and alloys. We will briefly discuss the stress–strain behavior of polymers as well.

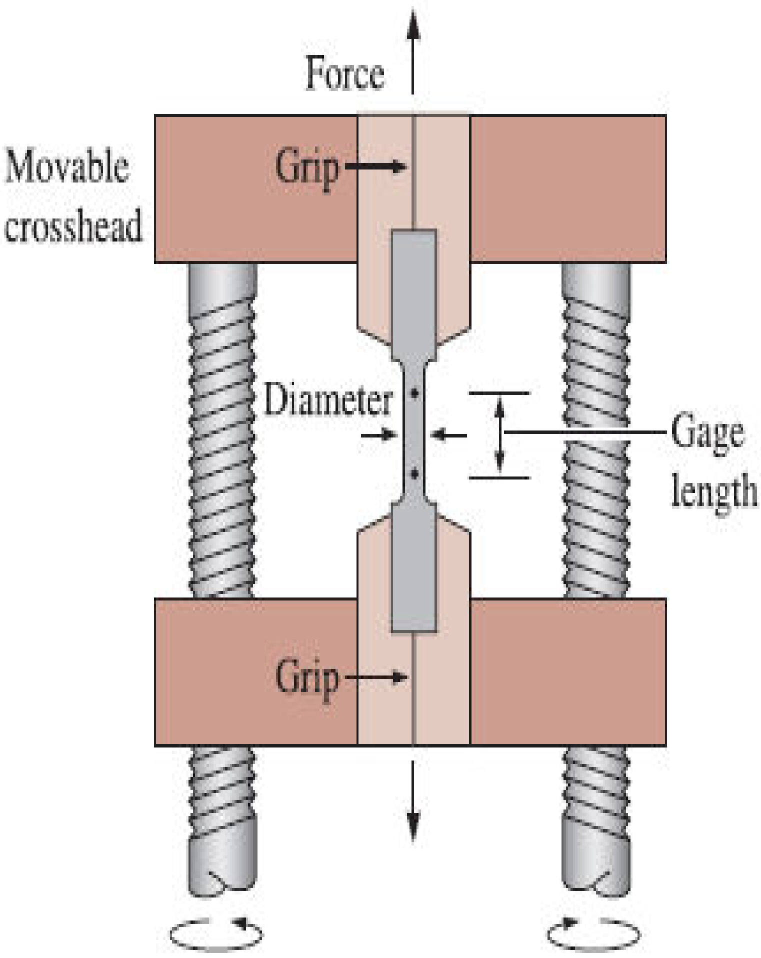


Figure 6-5
A unidirectional force is applied to a specimen in the tensile test by means of the moveable crosshead. The crosshead movement can be performed using screws or a hydraulic mechanism.

Figure 6-6 shows *qualitatively* the stress–strain curves for a typical (a) metal, (b) thermoplastic material, (c) elastomer, and (d) ceramic (or glass) under relatively small strain rates. The scales in this figure are qualitative and different for each material. In practice, the actual magnitude of stresses and strains will be very different. The temperature of the plastic material is assumed to be above its **glass-transition temperature** (T_g). The temperature of the metal is assumed to be room temperature. Metallic and thermoplastic materials show an initial elastic region followed by a non-linear plastic region. A separate curve for elastomers (e.g., rubber or silicones) is also included since the behavior of these materials is different from other polymeric materials. For elastomers, a large portion of the deformation is elastic and nonlinear. On the other hand, ceramics and glasses show only a linear elastic region and almost no plastic deformation at room temperature.

When a tensile test is conducted, the data recorded includes load or force as a function of change in length (Δl) as shown in Table 6-1 for an aluminum alloy test bar. These data are then subsequently converted into stress and strain. The stress-strain curve is analyzed further to extract properties of materials (e.g., Young's modulus, yield strength, etc.).

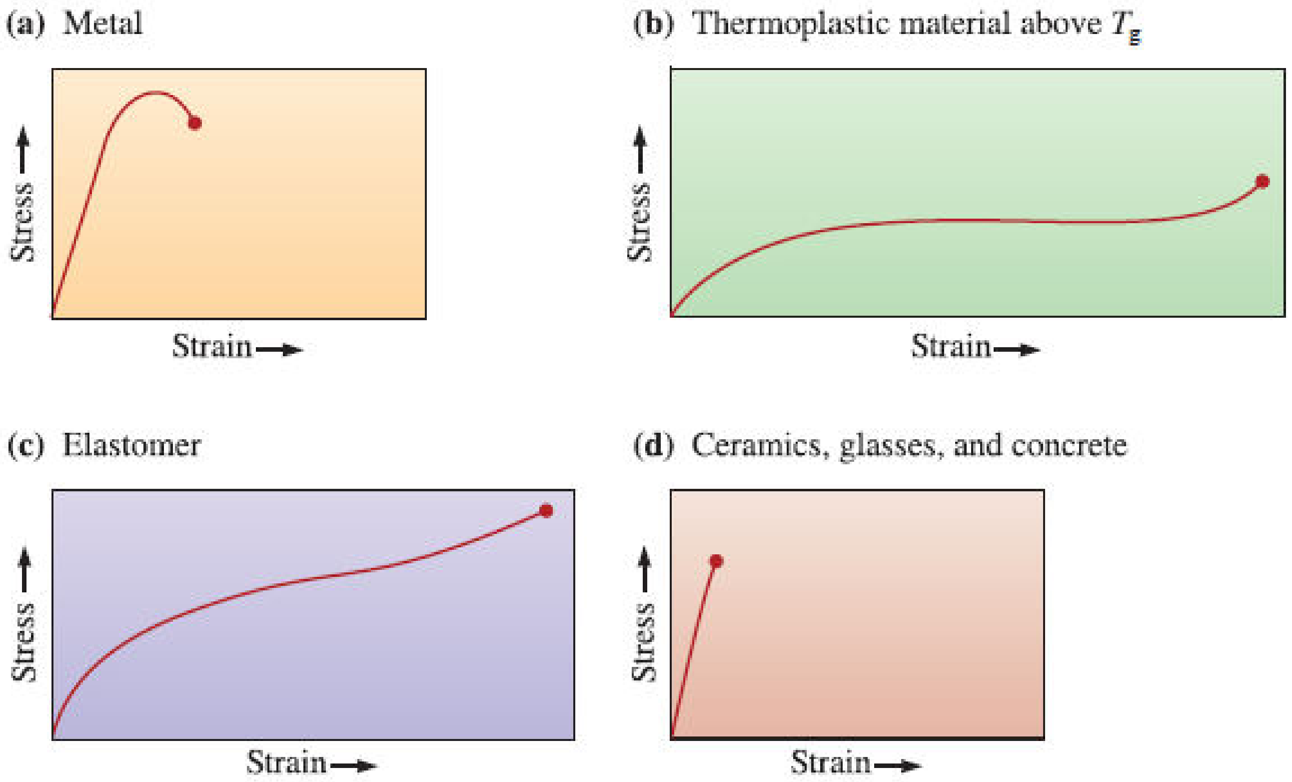


Figure 6-6 Tensile stress–strain curves for different materials. Note that these are *qualitative*. The magnitudes of the stresses and strains should not be compared.

TABLE 6-1 ■ ***The results of a tensile test of a 0.505-in. diameter aluminum alloy test bar, initial length (l_0) = 2 in.***

Load (lb)	Change in Length (in.)	Calculated Stress (psi)	Calculated Strain (in./in.)
0	0.000	0	0
1000	0.001	4,993	0.0005
3000	0.003	14,978	0.0015
5000	0.005	24,963	0.0025
7000	0.007	34,948	0.0035
7500	0.030	37,445	0.0150
7900	0.080	39,442	0.0400
8000 (maximum load)	0.120	39,941	0.0600
7950	0.160	39,691	0.0800
7600 (fracture)	0.205	37,944	0.1025

Engineering Stress and Strain The results of a single test apply to all sizes and cross-sections of specimens for a given material if we convert the force to stress and the distance between gage marks to strain. **Engineering stress** and **engineering strain** are defined by the following equations:

$$\text{Engineering stress} = S = \frac{F}{A_0} \tag{6-6}$$

$$\text{Engineering stress} = e = \frac{\Delta l}{l_0} \tag{6-7}$$

where A_0 is the *original* cross-sectional area of the specimen before the test begins, l_0 is the *original* distance between the gage marks, and Δl is the change in length after force F is applied. The conversions from load and sample length to stress and strain are included in Table 6-1. The stress-strain curve (Figure 6-7) is used to record the results of a tensile test.

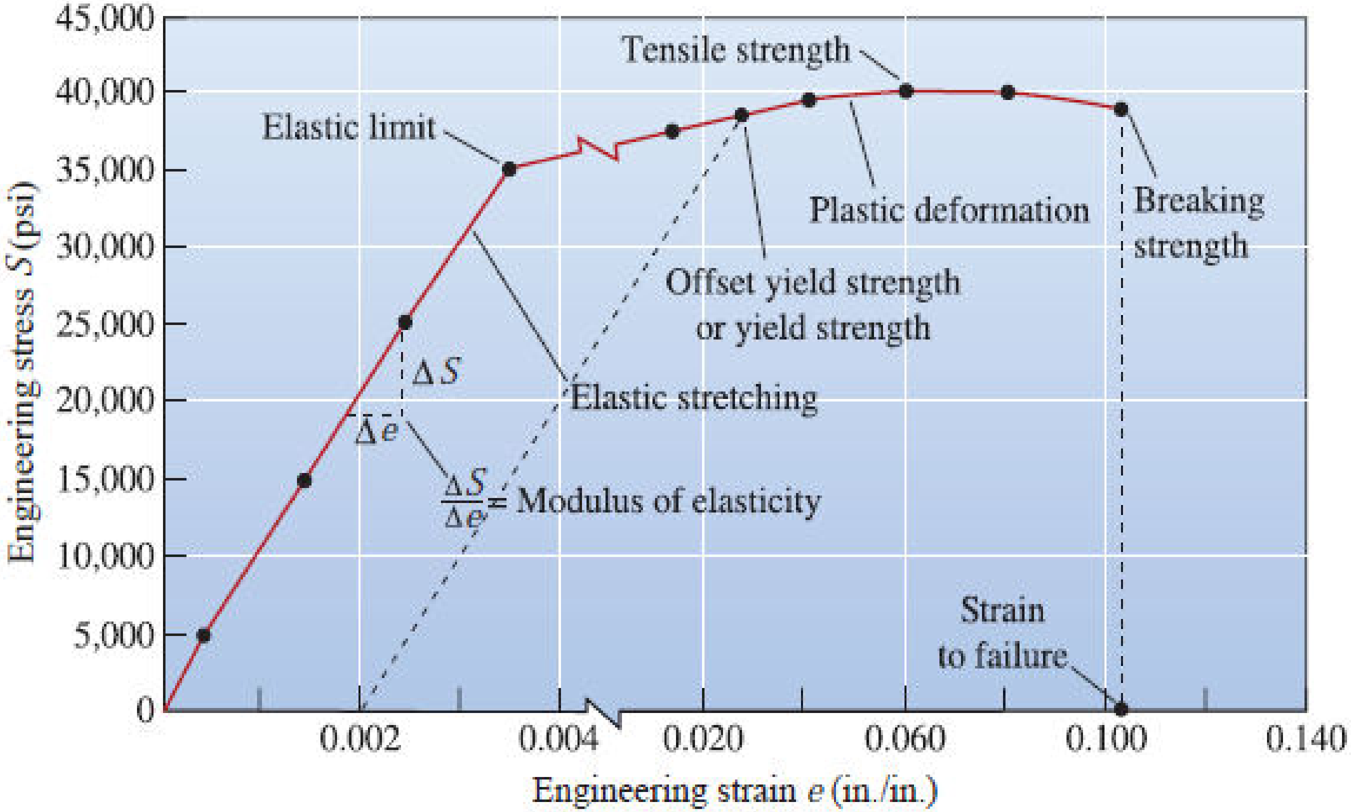


Figure 6-7 The engineering stress–strain curve for an aluminum alloy from Table 6-1.

Example 6-1 *Tensile Testing of Aluminum Alloy*

Convert the change in length data in Table 6-1 to engineering stress and strain and plot a stress-strain curve.

SOLUTION

For the 1000-lb load:

$$S = \frac{F}{A_0} = \frac{1000\text{ lb}}{(\pi/4)(0.505\text{ in})^2} = 4{,}993\text{ psi}$$

$$e = \frac{\Delta l}{l_0} = \frac{0.001\text{ in.}}{2.000\text{ in.}} = 0.0005\text{ in./in.}$$

The results of similar calculations for each of the remaining loads are given in Table 6-1 and are plotted in Figure 6-7.

Units Many different units are used to report the results of the tensile test. The most common units for stress are pounds per square inch (psi) and MegaPascals (MPa). The units for strain include inch/inch, centimeter/centimeter, and meter/meter, and thus, strain is often written as unitless. The conversion factors for stress are summarized in Table 6-2. Because strain is dimensionless, no conversion factors are required to change the system of units.

TABLE 6-2 ■ ***Units and conversion factors***

1 pound (lb) = 4.448 Newtons (N)
1 psi = pounds per square inch
1 MPa = MegaPascal = MegaNewtons per square meter (MN/m^2)
= Newtons per square millimeter (N/mm^2) = 10^6 Pa
1 GPa = 1000 MPa = GigaPascal
1 ksi = 1000 psi = 6.895 MPa
1 psi = 0.006895 MPa
1 MPa = 0.145 ksi = 145 psi

Properties Obtained from the Tensile Test

Yield Strength

As we apply stress to a material, the material initially exhibits elastic deformation. The strain that develops is completely recovered when the applied stress is removed. As we continue to increase the applied stress, the material eventually "yields" to the applied stress and exhibits both elastic and plastic deformation. The critical stress value needed to initiate plastic deformation is defined as the **elastic limit** of the material. In metallic materials, this is usually the stress required for dislocation motion, or slip, to be initiated. In polymeric materials, this stress will correspond to disentanglement of polymer molecule chains or sliding of chains past each other. The **proportional limit** is defined as the level of stress above which the relationship between stress and strain is not linear.

In most materials, the elastic limit and proportional limit are quite close; however, neither the elastic limit nor the proportional limit values can be determined precisely. Measured values depend on the sensitivity of the equipment used. We, therefore, define them at an **offset strain value** (typically, but not always, 0.002 or 0.2%). We then draw a line parallel to the linear portion of the engineering stress-strain curve starting at this offset value of strain. The stress value corresponding to the intersection of this line and the engineering stress-strain curve is defined as the **offset yield strength**, also often stated as the **yield strength**. The 0.2% offset yield strength for gray cast iron is 40,000 psi as shown in Figure 6-8(a). Engineers normally prefer to use the offset yield strength for design purposes because it can be reliably determined.

For some materials, the transition from elastic deformation to plastic flow is rather abrupt. This transition is known as the **yield point phenomenon**. In these materials,

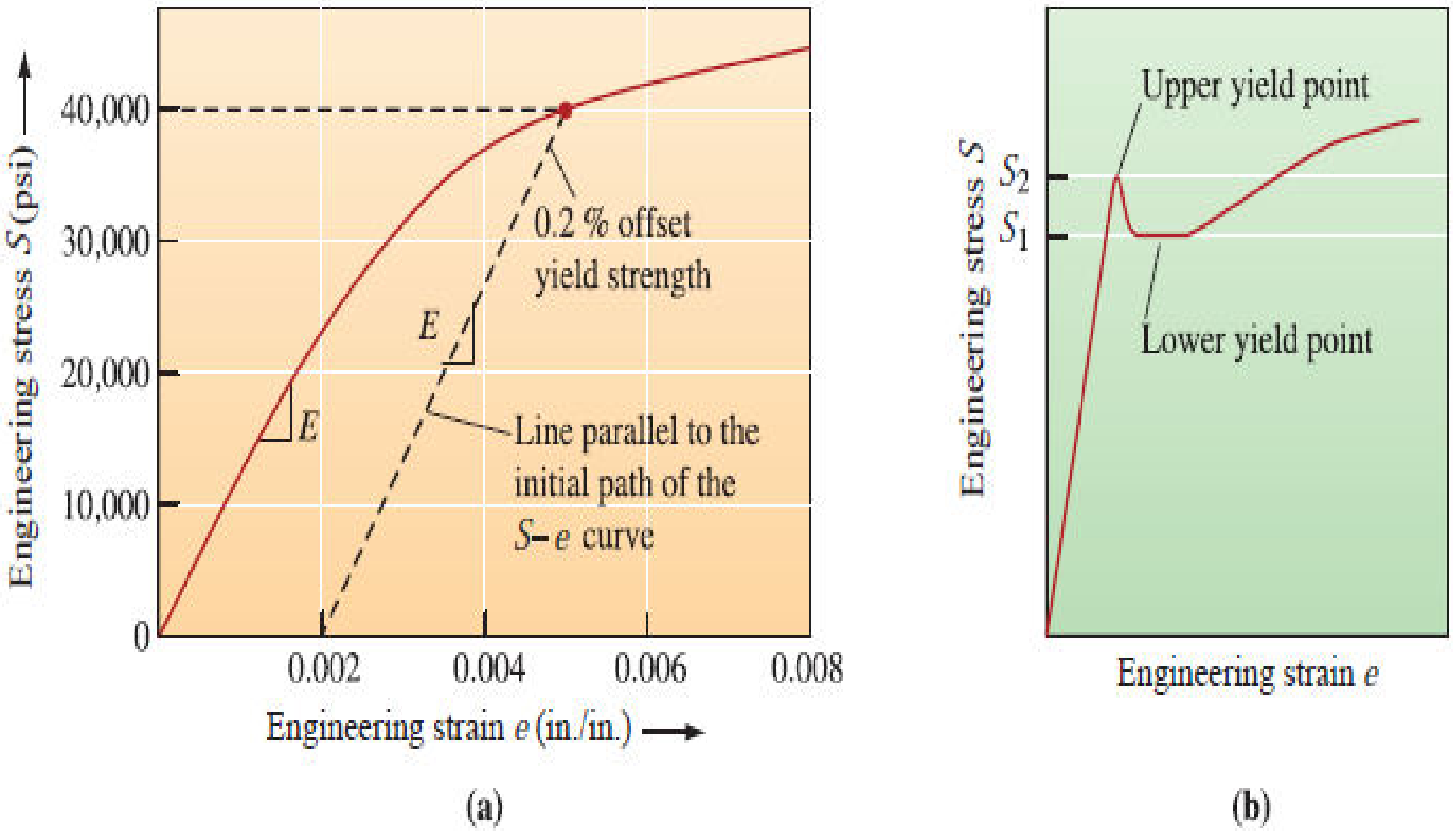


Figure 6-8 (a) Determining the 0.2% offset yield strength in gray cast iron, and (b) upper and lower yield point behavior in a low carbon steel.

as plastic deformation begins, the stress value drops first from the *upper yield point* (S_2) [Figure 6-8(b)]. The stress value then oscillates around an average value defined as the *lower yield point* (S_1). For these materials, the yield strength is usually defined from the 0.2% strain offset.

The stress-strain curve for certain low-carbon steels displays the yield point phenomenon [Figure 6-8(b)]. The material is expected to plastically deform at stress S_1; however, small interstitial atoms clustered around the dislocations interfere with slip and raise the yield point to S_2. Only after we apply the higher stress S_2 do the dislocations slip. After slip begins at S_2, the dislocations move away from the clusters of small atoms and continue to move very rapidly at the lower stress S_1.

When we design parts for load-bearing applications, we prefer little or no plastic deformation. As a result, we must select a material such that the design stress is considerably lower than the yield strength at the temperature at which the material will be used. We can also make the component cross-section larger so that the applied force produces a stress that is well below the yield strength. On the other hand, when we want to shape materials into components (e.g., take a sheet of steel and form a car chassis), we need to apply stresses that are well above the yield strength.

Tensile Strength The stress obtained at the highest applied force is the **tensile strength** (S_{UTS}), which is the maximum stress on the engineering stress-strain curve. This value is also commonly known as the **ultimate tensile strength**. In many ductile materials, deformation does not remain uniform. At some point, one region deforms more than others and a large local decrease in the cross-sectional area occurs (Figure 6-9). This locally deformed region is called a "neck." This phenomenon is known as **necking**. Because the cross-sectional area becomes smaller at this point, a lower force is required to continue its deformation, and the engineering stress, calculated from the *original* area A_0, decreases. The tensile strength is the stress at which necking begins in ductile metals. In compression testing, the materials will bulge; thus necking is seen only in a tensile test.

Figure 6-10 shows typical yield strength values for various engineering materials. Ultra-pure metals have a yield strength of ~ 1 – 10 MPa. On the other hand, the yield strength of alloys is higher. Strengthening in alloys is achieved using different mechanisms

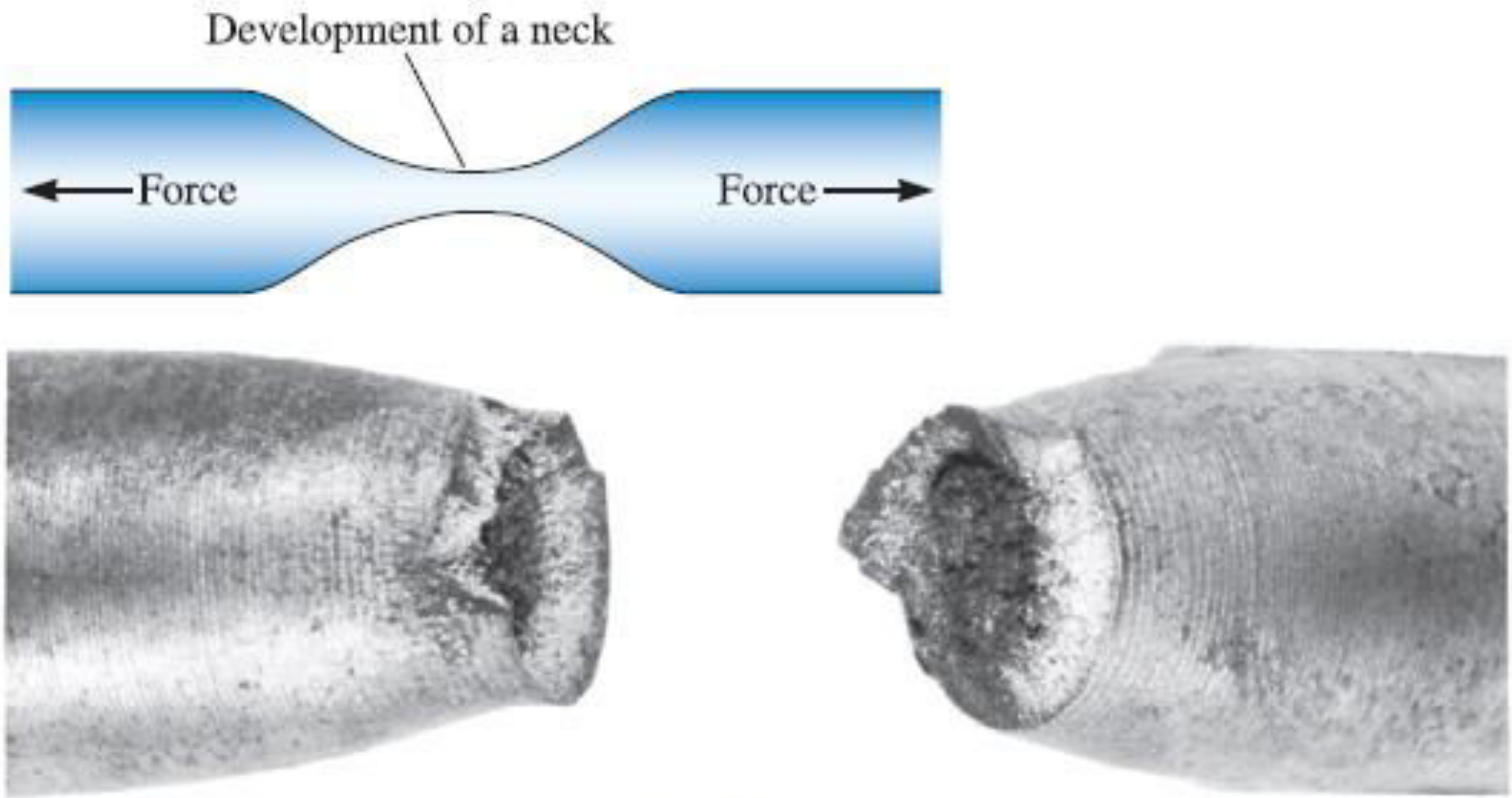


Figure 6-9 Localized deformation of a ductile material during a tensile test produces a necked region. The micrograph shows a necked region in a fractured sample. (*This article was published in Materials Principles and Practice, Charles Newey and Graham Weaver (Eds.), Figure 6.9, p. 300, Coyright Open University.*)

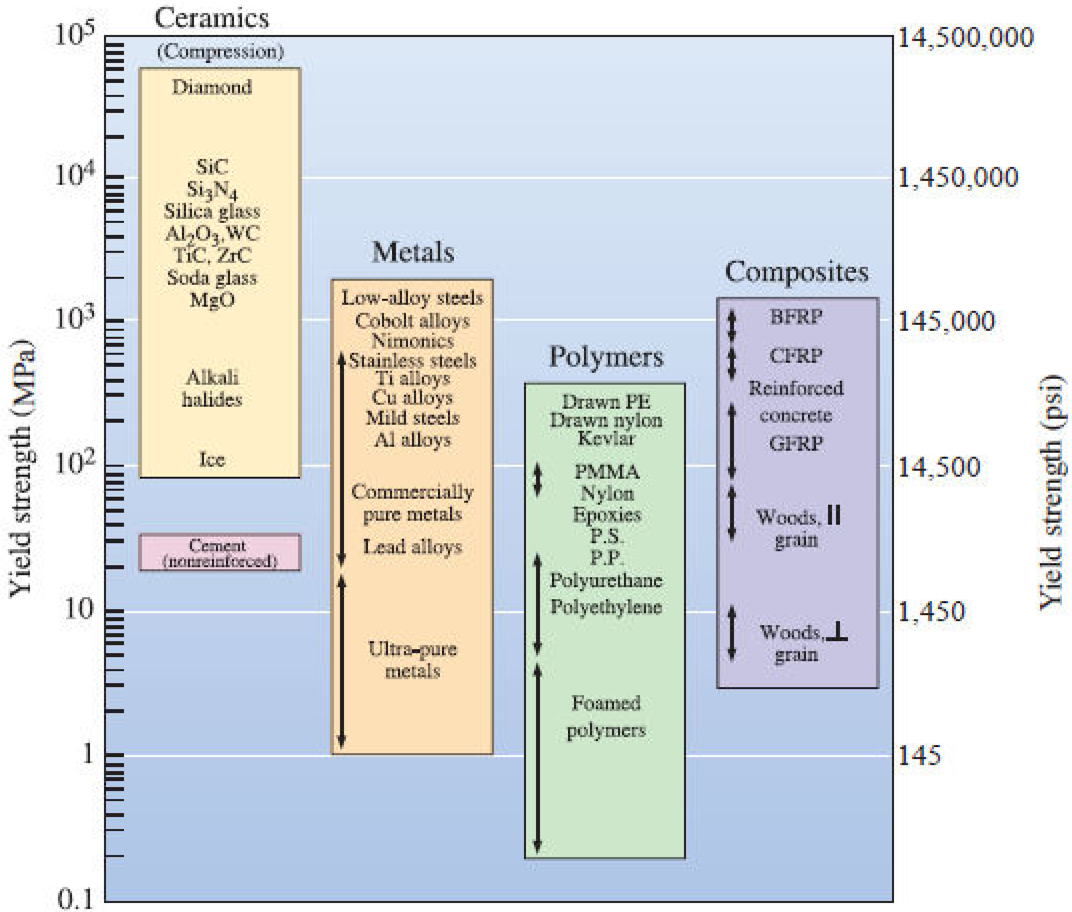


Figure 6-10 Typical yield strength values for different engineering materials. Note that values shown are in MPa and psi. (*Reprinted from* Engineering Materials I, *Second Edition, M.F. Ashby and D.R.H. Jones, 1996, Fig. 8-12, p. 85. Copyright © 1996 Butterworth-Heinemann. Reprinted with permission from Elsevier Science.*)

described before (e.g., grain size, solid solution formation, strain hardening, etc.). The yield strength of a particular metal or alloy is usually the same for tension and compression. The yield strength of plastics and elastomers is generally lower than metals and alloys, ranging up to about 10 – 100 MPa. The values for ceramics (Figure 6-10) are for compressive strength (obtained using a hardness test). Tensile strength of most ceramics is much lower (~100–200 MPa). The tensile strength of glasses is about ~70 MPa and depends on surface flaws.

Elastic Properties The modulus of elasticity, or *Young's modulus* (E), is the slope of the stress-strain curve in the elastic region. This relationship between stress and strain in the elastic region is known as **Hooke's Law**:

$$E = \frac{S}{e} \tag{6-8}$$

The modulus is closely related to the binding energies of the atoms. (Figure 2-18). A steep slope in the force-distance graph at the equilibrium spacing (Figure 2-19) indicates that high forces are required to separate the atoms and cause the material to stretch elastically. Thus, the material has a high modulus of elasticity. Binding forces, and thus the modulus of elasticity, are typically higher for high melting point materials (Table 6-3). In metallic materials, the modulus of elasticity is considered a microstructure *insensitive* property since the value is dominated by the stiffness of atomic bonds. Grain size or other microstructural features do not have a very large effect on the Young's modulus. Note that Young's modulus does depend on such factors as orientation of a single crystal material (i.e., it depends upon crystallographic direction). For ceramics, the Young's modulus depends on the level of porosity. The Young's modulus of a composite depends upon the stiffness and amounts of the individual components.

The **stiffness** of a component is proportional to its Young's modulus. (The stiffness also depends on the component dimensions.) A component with a high modulus of elasticity will show much smaller changes in dimensions if the applied stress causes only elastic deformation when compared to a component with a lower elastic modulus. Figure 6-11 compares the elastic behavior of steel and aluminum. If a stress of 30,000 psi is applied to each material, the steel deforms elastically 0.001 in./in.; at the same stress, aluminum deforms 0.003 in./in. The elastic modulus of steel is about three times higher than that of aluminum.

Figure 6-12 shows the ranges of elastic moduli for various engineering materials. The modulus of elasticity of plastics is much smaller than that for metals or ceramics and glasses. For example, the modulus of elasticity of nylon is 2.7 GPa ($\sim 0.4 \times 10^6$ psi); the modulus of glass fibers is 72 GPa ($\sim 10.5 \times 10^6$ psi). The Young's modulus of composites

TABLE 6-3 *Elastic properties and melting temperature (T_m) of selected materials*

Material	T_m (°C)	*E* (psi)	Poisson's ratio (ν)
Pb	327	2.0×10^6	0.45
Mg	650	6.5×10^6	0.29
Al	660	10.0×10^6	0.33
Cu	1085	18.1×10^6	0.36
Fe	1538	30.0×10^6	0.27
W	3410	59.2×10^6	0.28
Al_2O_3	2020	55.0×10^6	0.26
Si_3N_4		44.0×10^6	0.24

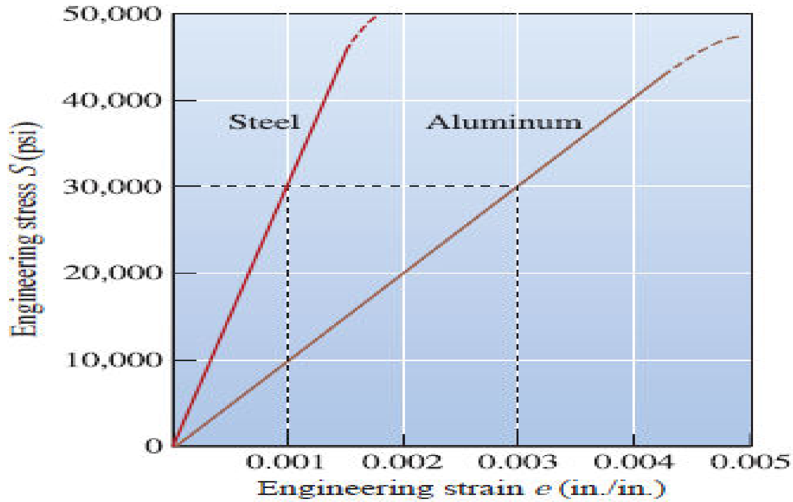


Figure 6-11
Comparison of the elastic behavior of steel and aluminum. For a given stress, aluminum deforms elastically three times as much as does steel (i.e., the elastic modulus of aluminum is about three times lower than that of steel).

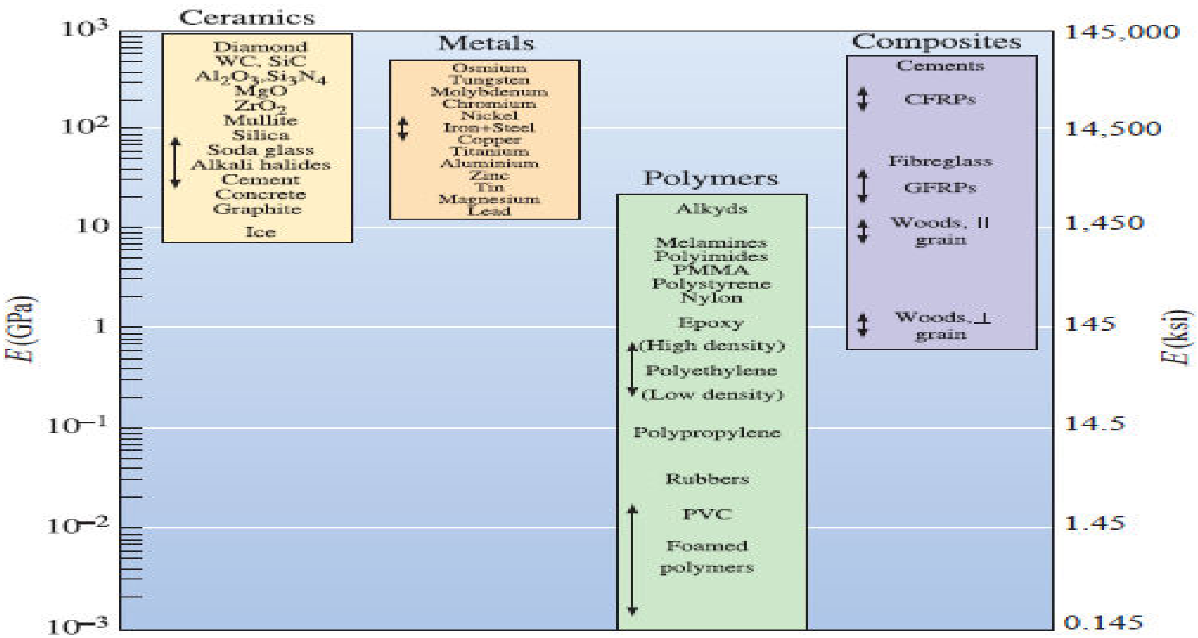


Figure 6-12 Range of elastic moduli for different engineering materials. Note: Values are shown in GPa and ksi. (*Reprinted from* Engineering Materials I, *Second Edition, M.F. Ashby and D.R.H. Jones, 1996, Fig. 3-5, p. 35, Copyright © 1996 Butterworth-Heinemann. Reprinted with permission from Elsevier Science.*)

such as glass fiber-reinforced composites (GFRC) or carbon fiber-reinforced composites (CFRC) lies between the values for the matrix polymer and the fiber phase (carbon or glass fibers) and depends upon their relative volume fractions. The Young's modulus of many alloys and ceramics is higher, generally ranging up to 410 GPa (~60,000 ksi). Ceramics, because of the strength of ionic and covalent bonds, have the highest elastic moduli.

Poisson's ratio, ν, relates the longitudinal elastic deformation produced by a simple tensile or compressive stress to the lateral deformation that occurs simultaneously:

$$\nu = \frac{-e_{\text{lateral}}}{e_{\text{longitudinal}}} \tag{6-9}$$

For many metals in the elastic region, the Poisson's ratio is typically about 0.3 (Table 6-3). During a tensile test, the ratio increases beyond yielding to about 0.5, since during plastic deformation, volume remains constant. Some interesting structures, such as some honeycomb structures and foams, exhibit negative Poisson's ratios. *Note: Poisson's ratio should not be confused with the kinematic viscosity, both of which are denoted by the Greek letter* ν.

The **modulus of resilience** (E_r), the area contained under the elastic portion of a stress-strain curve, is the elastic energy that a material absorbs during loading and subsequently releases when the load is removed. For linear elastic behavior:

$$E_r = \left(\frac{1}{2}\right)(\text{yield strength})(\text{strain at yielding}) \tag{6-10}$$

The ability of a spring or a golf ball to perform satisfactorily depends on a high modulus of resilience.

True Stress and True Strain

The decrease in engineering stress beyond the tensile strength on an engineering stress–strain curve is related to the definition of engineering stress. We used the original area A_0 in our calculations, but this is not precise because the area continually changes. We define **true stress** and **true strain** by the following equations:

$$\text{True stress} = \sigma = \frac{F}{A} \tag{6-13}$$

$$\text{True strain} = \varepsilon = \int_{l_0}^{l} \frac{dl}{l} = \ln\left(\frac{l}{l_0}\right) \tag{6-14}$$

where A is the instantaneous area over which the force F is applied, l is the instantaneous sample length, and l_0 is the initial length. In the case of metals, plastic deformation is essentially a constant-volume process (i.e., the creation and propagation of dislocations results in a negligible volume change in the material). When the constant volume assumption holds, we can write

$$A_0 l_0 = Al \text{ or } A = \frac{A_0 l_0}{l} \tag{6-15}$$

and using the definitions of engineering stress S and engineering strain e, Equation 6-13 can be written as

$$\sigma = \frac{F}{A} = \frac{F}{A_0}\left(\frac{l}{l_0}\right) = S\left(\frac{l_0 + \Delta l}{l_0}\right) = S(1 + e) \tag{6-16}$$

It can also be shown that

$$\varepsilon = \ln(1 + e) \tag{6-17}$$

Thus, it is a simple matter to convert between the engineering stress–strain and true stress–strain systems. Note that the expressions in Equations 6-16 and 6-17 are not valid after the onset of necking, because after necking begins, the distribution of strain along the gage length is not uniform. After necking begins, Equation 6-13 must be used to calculate the true stress and the expression

$$\varepsilon = \ln\left(\frac{A_0}{A}\right) \tag{6-18}$$

must be used to calculate the true strain. Equation 6-18 follows from Equations 6-14 and 6-15. After necking the instantaneous area A is the cross-sectional area of the neck A_{neck}.

The true stress–strain curve is compared to the engineering stress–strain curve in Figure 6-14(a). There is no maximum in the true stress–true strain curve.

Note that it is difficult to measure the instantaneous cross-sectional area of the neck. Thus, true stress–strain curves are typically truncated at the true stress that corresponds to the ultimate tensile strength, as shown in Figure 6-14(b).

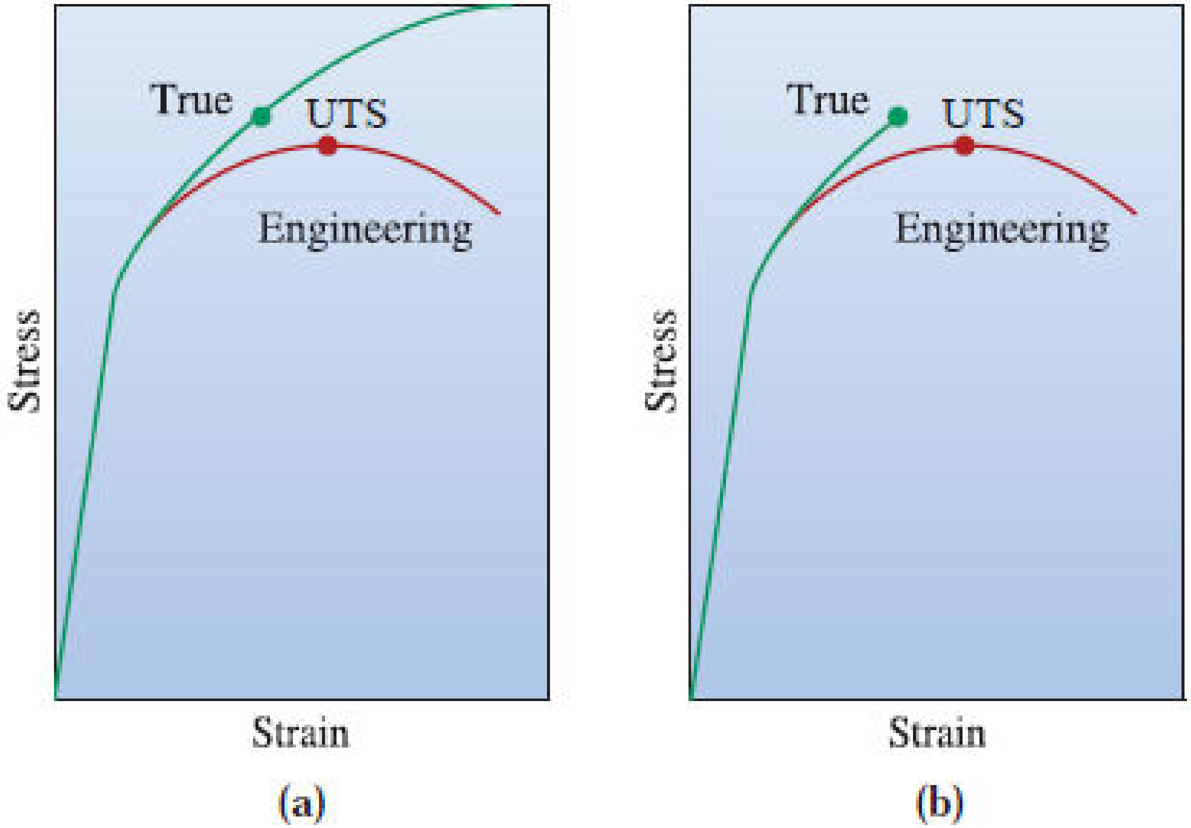


Figure 6-14 (a) The relation between the true stress–true strain diagram and engineering stress–engineering strain diagram. The curves are nominally identical to the yield point. The true stress corresponding to the ultimate tensile strength (UTS) is indicated. (b) Typically true stress–strain curves must be truncated at the true stress corresponding to the ultimate tensile strength, since the cross-sectional area at the neck is unknown.

Example 6-5 *True Stress and True Strain*

Compare engineering stress and strain with true stress and strain for the aluminum alloy in Example 6-1 at (a) the maximum load and (b) fracture. The diameter at maximum load is 0.4905 in. and at fracture is 0.398 in.

SOLUTION

(a) At the maximum load,

$$\text{Engineering stress } S = \frac{F}{A_0} = \frac{8000 \text{ lb}}{(\pi/4)(0.505 \text{ in})^2} = 39{,}941 \text{ psi}$$

$$\text{Engineering strain } e = \frac{\Delta l}{l_0} = \frac{2.120 - 2.000}{2.000} = 0.060 \text{ in./in.}$$

$$\text{True stress} = \sigma = S(1 + e) = 39{,}941(1 + 0.060) = 42{,}337 \text{ psi}$$

$$\text{True strain} = \ln(1 + e) = \ln(1 + 0.060) = 0.058 \text{ in./in.}$$

The maximum load is the last point at which the expressions used here for true stress and true strain apply. Note that the same answers are obtained for true stress and strain if the instantaneous dimensions are used:

$$\sigma = \frac{F}{A} = \frac{8000 \text{ lb}}{(\pi/4)(0.4905 \text{ in})^2} = 42{,}337 \text{ psi}$$

$$\varepsilon = \ln\left(\frac{A_0}{A}\right) = \ln\left[\frac{(\pi/4)(0.505 \text{ in}^2)}{(\pi/4)(0.4905 \text{ in}^2)}\right] = 0.058 \text{ in./in.}$$

Up until the point of necking in a tensile test, the engineering stress is less than the corresponding true stress, and the engineering strain is greater than the corresponding true strain.

(b) At fracture,

$$S = \frac{F}{A_0} = \frac{7600 \text{ lb}}{(\pi/4)(0.505 \text{ in})^2} = 37{,}944 \text{ psi}$$

$$e = \frac{\Delta l}{l_0} = \frac{2.205 - 2.000}{2.000} = 0.103 \text{ in./in.}$$

$$\sigma = \frac{F}{A_f} = \frac{7600 \text{ lb}}{(\pi/4)(0.398 \text{ in})^2} = 61{,}088 \text{ psi}$$

$$\varepsilon = \ln\left(\frac{A_0}{A_f}\right) = \ln\left[\frac{(\pi/4)(0.505 \text{ in}^2)}{(\pi/4)(0.398 \text{ in}^2)}\right] = \ln(1.601) = 0.476 \text{ in./in.}$$

It was necessary to use the instantaneous dimensions to calculate the true stress and strain, since failure occurs past the point of necking. After necking, the true strain is greater than the corresponding engineering strain.

The Bend Test for Brittle Materials

In ductile metallic materials, the engineering stress–strain curve typically goes through a maximum; this maximum stress is the tensile strength of the material. Failure occurs at a lower engineering stress after necking has reduced the cross-sectional area supporting the load. In more brittle materials, failure occurs at the maximum load, where the tensile strength and breaking strength are the same (Figure 6-15).

In many brittle materials, the normal tensile test cannot easily be performed because of the presence of flaws at the surface. Often, just placing a brittle material in the grips of the tensile testing machine causes cracking. These materials may be tested using the **bend test** [Figure 6-16(a)]. By applying the load at three points and causing bending, a tensile force acts on the material opposite the midpoint. Fracture begins at this location. The **flexural strength**, or **modulus of rupture**, describes the material's strength:

$$\text{Flexural strength for three-point bend test } \sigma_{\text{bend}} = \frac{3FL}{2wh^2} \qquad (6\text{-}19)$$

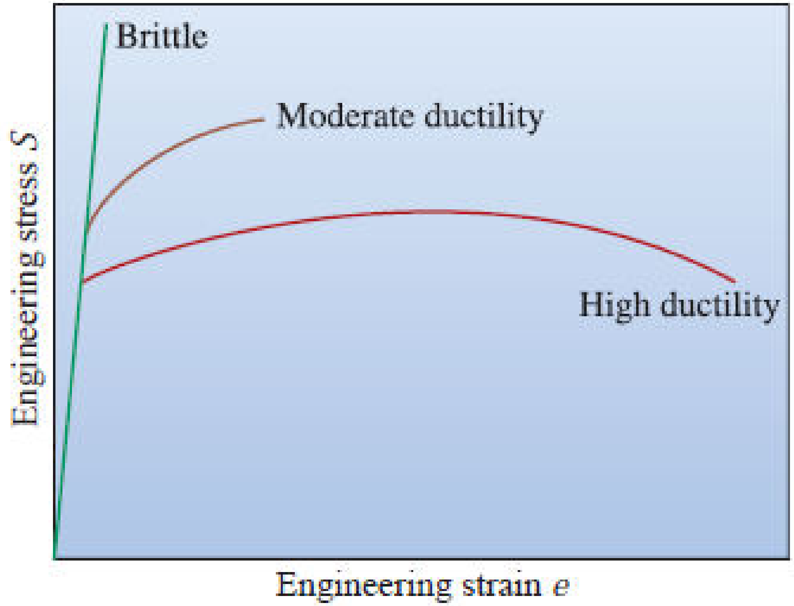


Figure 6-15 The engineering stress–strain behavior of brittle materials compared with that of more ductile materials.

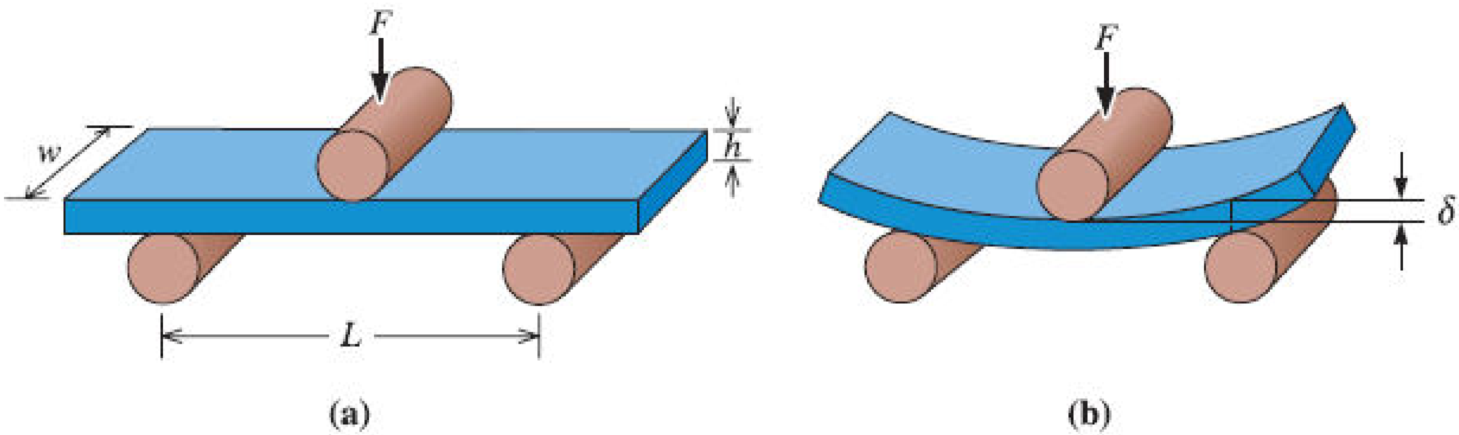


Figure 6-16 (a) The bend test often used for measuring the strength of brittle materials, and (b) the deflection δ obtained by bending.

where F is the fracture load, L is the distance between the two outer points, w is the width of the specimen, and h is the height of the specimen. The flexural strength has units of stress. The results of the bend test are similar to the stress-strain curves; however, the stress is plotted versus deflection rather than versus strain (Figure 6-17). The corresponding bending moment diagram is shown in Figure 6-18(a).

The modulus of elasticity in bending, or the **flexural modulus** (E_{bend}), is calculated as

$$\text{Flexural modulus } E_{bend} = \frac{L^3F}{4wh^3\delta} \tag{6-20}$$

where δ is the deflection of the beam when a force F is applied.

This test can also be conducted using a setup known as the four-point bend test [Figure 6-18(b)]. The maximum stress or flexural stress for a four-point bend test is given by

$$\sigma_{bend} = \frac{3FL_1}{4wh^2} \tag{6-21}$$

for the specific case in which $L_1 = L/4$ in Figure 6-18(b).

Note that the derivations of Equations 6-19 through 6-21 assume a linear stress–strain response (and thus cannot be correctly applied to many polymers). The four-point bend

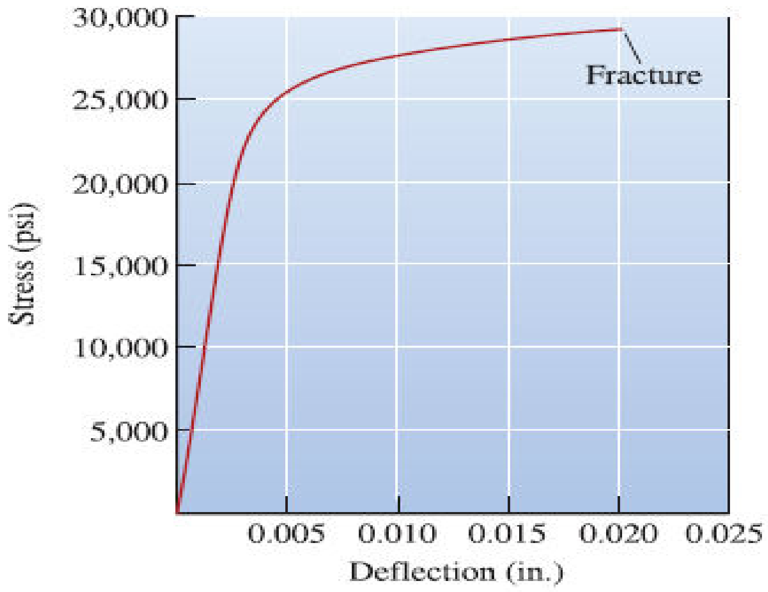


Figure 6-17 Stress-deflection curve for an MgO ceramic obtained from a bend test.

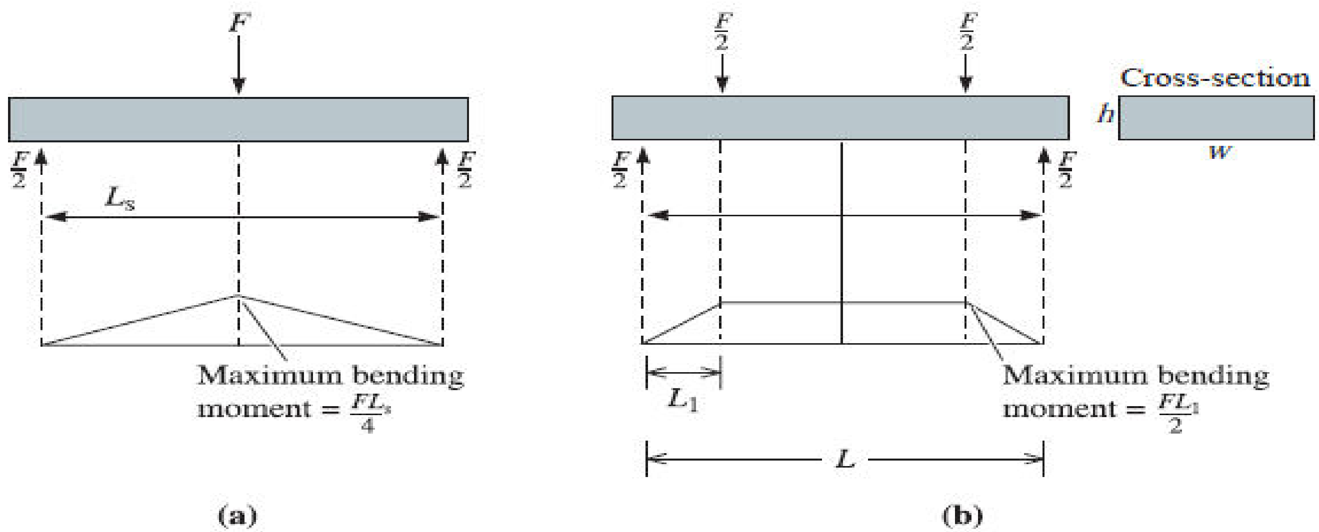


Figure 6-18 (a) Three-point and (b) four-point bend test setup.

test is better suited for testing materials containing flaws. This is because the bending moment between the inner platens is constant [Figure 6-18(b)]; thus samples tend to break randomly unless there is a flaw that locally raises the stress.

Since cracks and flaws tend to remain closed in compression, brittle materials such as concrete are often incorporated into designs so that only compressive stresses act on the part. Often, we find that brittle materials fail at much higher compressive stresses than tensile stresses (Table 6-4). This is why it is possible to support a fire truck on four coffee cups; however, ceramics have very limited mechanical toughness. Hence, when we drop a ceramic coffee cup, it can break easily.

TABLE 6-4 ■ *Comparison of the tensile, compressive, and flexural strengths of selected ceramic and composite materials*

Material	Tensile Strength (psi)	Compressive Strength (psi)	Flexural Strength (psi)
Polyester—50% glass fibers	23,000	32,000	45,000
Polyester—50% glass fiber fabric	37,000	27,000[a]	46,000
Al_2O_3 (99% pure)	30,000	375,000	50,000
SiC (pressureless-sintered)	25,000	560,000	80,000

[a] *A number of composite materials are quite poor in compression.*

Ductility

Ductility is the ability of a material to be permanently deformed without breaking when a force is applied. There are two common measures of ductility. The **percent elongation** quantifies the permanent plastic deformation at failure (i.e., the elastic deformation recovered after fracture is not included) by measuring the distance between gage marks on the specimen before and after the test. Note that the strain after failure is smaller than the strain at the breaking point, because the elastic strain is recovered when the load is removed. The percent elongation can be written as

$$\%\ \text{Elongation} = \frac{l_f - l_0}{l_0} \times 100 \tag{6-11}$$

where l_f is the distance between gage marks after the specimen breaks.

A second approach is to measure the percent change in the cross-sectional area at the point of fracture before and after the test. The **percent reduction in area** describes the amount of thinning undergone by the specimen during the test:

$$\%\ \text{Reduction in area} = \frac{A_0 - A_f}{A_0} \times 100 \tag{6-12}$$

where A_f is the final cross-sectional area at the fracture surface.

Ductility is important to both designers of load-bearing components and manufacturers of components (bars, rods, wires, plates, I-beams, fibers, etc.) utilizing materials processing.

Example 6-4 *Ductility of an Aluminum Alloy*

The aluminum alloy in Example 6-1 has a final length after failure of 2.195 in. and a final diameter of 0.398 in. at the fractured surface. Calculate the ductility of this alloy.

SOLUTION

$$\% \text{ Elongation} = \frac{l_f - l_0}{l_0} \times 100 = \frac{2.195 - 2.000}{2.000} \times 100 = 9.75\%$$

$$\% \text{ Reduction in area} = \frac{A_0 - A_f}{A_0} \times 100$$

$$= \frac{(\pi/4)(0.505)^2 - (\pi/4)(0.398)^2}{(\pi/4)(0.505)^2} \times 100$$

$$= 37.9\%$$

The final length is less than 2.205 in. (see Table 6-1) because, after fracture, the elastic strain is recovered.

TABLE 6-1 ■ ***The results of a tensile test of a 0.505-in. diameter aluminum alloy test bar, initial length (l_0) = 2 in.***

Load (lb)	Change in Length (in.)	Calculated Stress (psi)	Calculated Strain (in./in.)
0	0.000	0	0
1000	0.001	4,993	0.0005
3000	0.003	14,978	0.0015
5000	0.005	24,963	0.0025
7000	0.007	34,948	0.0035
7500	0.030	37,445	0.0150
7900	0.080	39,442	0.0400
8000 (maximum load)	0.120	39,941	0.0600
7950	0.160	39,691	0.0800
7600 (fracture)	0.205	37,944	0.1025

Effect of Temperature Mechanical properties of materials depend on temperature (Figure 6-13). Yield strength, tensile strength, and modulus of elasticity decrease at higher temperatures, whereas ductility commonly increases. A materials fabricator may wish to deform a material at a high temperature (known as *hot working*) to take advantage of the higher ductility and lower required stress.

We use the term "high temperature" here with a note of caution. A high temperature is defined relative to the melting temperature. Thus, 500°C is a high temperature for aluminum alloys; however, it is a relatively low temperature for the processing of steels. In metals, the yield strength decreases rapidly at higher temperatures due to a decreased dislocation density and an increase in grain size via grain growth (Chapter 5) or a related process known as recrystallization (as described later in Chapter 8). Similarly, any strengthening that may have occurred due to the formation of ultrafine precipitates may also decrease as the precipitates begin to either grow in size or dissolve into the matrix. We will discuss these effects in greater detail in later chapters. When temperatures are reduced, many, but not all, metals and alloys become brittle.

Increased temperatures also play an important role in forming polymeric materials and inorganic glasses. In many polymer-processing operations, such as extrusion or the stretch-blow process (Chapter 16), the increased ductility of polymers at higher temperatures is advantageous. Again, a word of caution concerning the use of the term "high temperature." For polymers, the term "high temperature" generally means a temperature

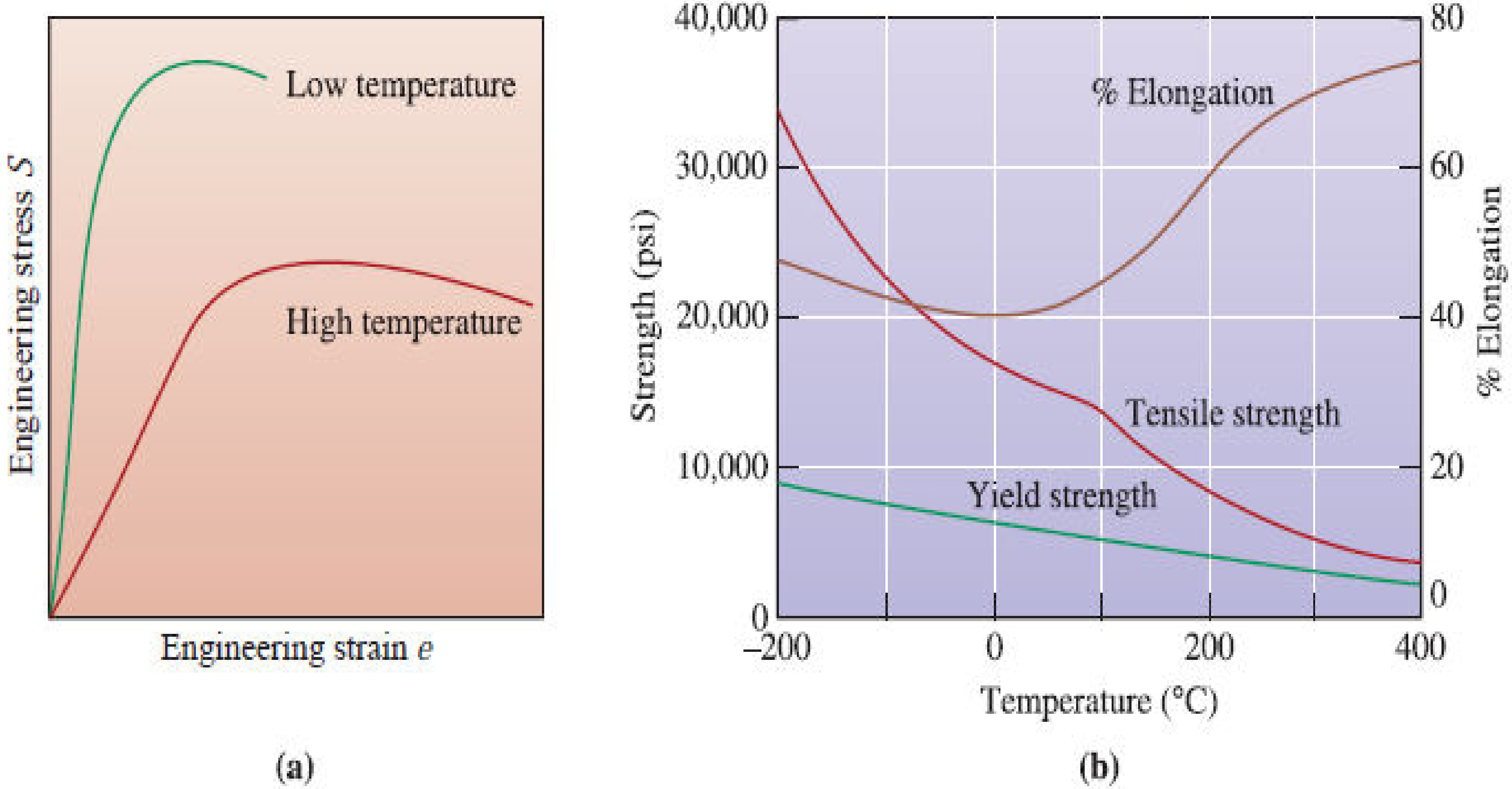


Figure 6-13 The effect of temperature (a) on the stress–strain curve and (b) on the tensile properties of an aluminum alloy.

higher than the glass-transition temperature (T_g). For our purpose, the glass-transition temperature is a temperature below which materials behave as brittle materials. Above the glass-transition temperature, plastics become ductile. The glass-transition temperature is not a fixed temperature, but depends on the rate of cooling as well as the polymer molecular weight distribution. Many plastics are ductile at room temperature because their glass-transition temperatures are *below* room temperature. To summarize, many polymeric materials will become harder and more brittle as they are exposed to temperatures that are below their glass-transition temperatures. The reasons for loss of ductility at lower temperatures in polymers and metallic materials are different; however, this is a factor that played a role in the failures of the *Titanic* in 1912 and the *Challenger* in 1986.

Ceramic and glassy materials are generally considered brittle at room temperature. As the temperature increases, glasses can become more ductile. As a result, glass processing (e.g., fiber drawing or bottle manufacturing) is performed at high temperatures.

Hardness of Materials

The **hardness test** measures the resistance to penetration of the surface of a material by a hard object. Hardness as a term is not defined precisely. Hardness, depending upon the context, can represent resistance to scratching or indentation and a qualitative measure of the strength of the material. In general, in **macrohardness** measurements, the load applied is ~2 N. A variety of hardness tests have been devised, but the most commonly used are

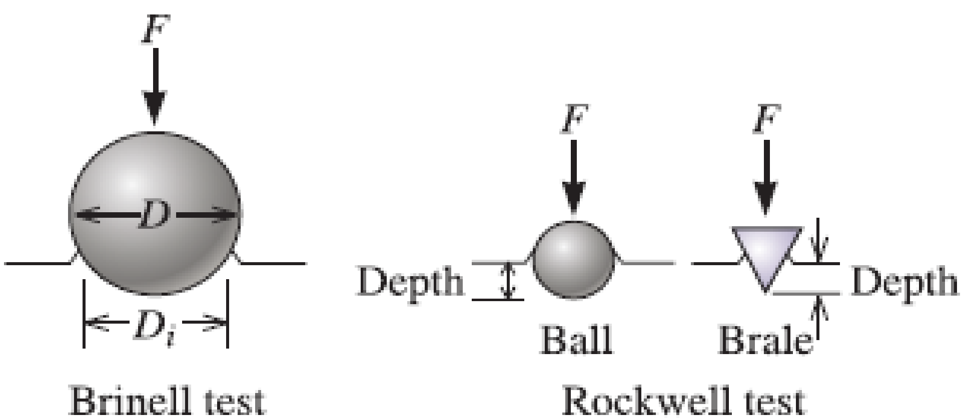
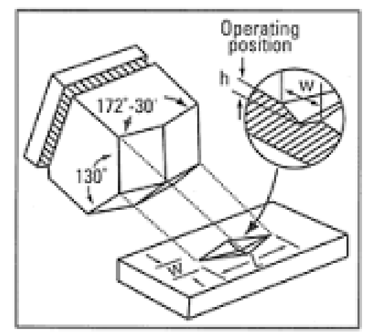


Figure 6-19
Indenters for the Brinell and Rockwell hardness tests.



the Rockwell test and the Brinell test. Different indenters used in these tests are shown in Figure 6-19.

In the *Brinell hardness test*, a hard steel sphere (usually 10 mm in diameter) is forced into the surface of the material. The diameter of the impression, typically 2 to 6 mm, is measured and the Brinell hardness number (abbreviated as HB or BHN) is calculated from the following equation:

$$HB = \frac{2F}{\pi D\left[D - \sqrt{D^2 - D_i^2}\right]} \qquad (6\text{-}22)$$

where F is the applied load in kilograms, D is the diameter of the indenter in millimeters, and D_i is the diameter of the impression in millimeters. The Brinell hardness has units of kg/mm^2.

The *Rockwell hardness test* uses a small-diameter steel ball for soft materials and a diamond cone, or Brale, for harder materials. The depth of penetration of the indenter is automatically measured by the testing machine and converted to a Rockwell hardness number (HR). Since an optical measurement of the indentation dimensions is not needed, the Rockwell test tends to be more popular than the Brinell test. Several variations of the Rockwell test are used, including those described in Table 6-5. A Rockwell *C* (HRC) test is used for hard steels, whereas a Rockwell *F* (HRF) test might be selected for aluminum. Rockwell tests provide a hardness number that has no units.

Hardness numbers are used primarily as a *qualitative* basis for comparison of materials, specifications for manufacturing and heat treatment, quality control, and

TABLE 6-5 ■ *Comparison of typical hardness tests*

Test	Indenter	Load	Application
Brinell	10-mm ball	3000 kg	Cast iron and steel
Brinell	10-mm ball	500 kg	Nonferrous alloys
Rockwell *A*	Brale	60 kg	Very hard materials
Rockwell *B*	1/16-in. ball	100 kg	Brass, low-strength steel
Rockwell *C*	Brale	150 kg	High-strength steel
Rockwell *D*	Brale	100 kg	High-strength steel
Rockwell *E*	1/8-in. ball	100 kg	Very soft materials
Rockwell *F*	1/16-in. ball	60 kg	Aluminum, soft materials
Vickers	Diamond square pyramid	10 kg	All materials
Knoop	Diamond elongated pyramid	500 g	All materials

correlation with other properties of materials. For example, Brinell hardness is related to the tensile strength of steel by the approximation:

$$\text{Tensile strength (psi)} = 500HB \tag{6-23}$$

where HB has units of kg/mm^2.

Hardness correlates well with wear resistance. A separate test is available for measuring the wear resistance. A material used in crushing or grinding of ores should be very hard to ensure that the material is not eroded or abraded by the hard feed materials. Similarly, gear teeth in the transmission or the drive system of a vehicle should be hard enough that the teeth do not wear out. Typically we find that polymer materials are exceptionally soft, metals and alloys have intermediate hardness, and ceramics are exceptionally hard. We use materials such as tungsten carbide-cobalt composite (WC-Co), known as "carbide," for cutting tool applications. We also use microcrystalline diamond or diamond-like carbon (DLC) materials for cutting tools and other applications.

The Knoop hardness (HK) test is a **microhardness test**, forming such small indentations that a microscope is required to obtain the measurement. In these tests, the load applied is less than 2 N. The Vickers test, which uses a diamond pyramid indenter, can be conducted either as a macro or microhardness test. Microhardness tests are suitable for materials that may have a surface that has a higher hardness than the bulk, materials in which different areas show different levels of hardness, or samples that are not macroscopically flat.

Strain Rate Effects and Impact Behavior

When a material is subjected to a sudden, intense blow, in which the strain rate ($\dot{\gamma}$ or $\dot{\varepsilon}$) is extremely rapid, it may behave in much more brittle a manner than is observed in the tensile test. This, for example, can be seen with many plastics and materials such as Silly Putty®. If you stretch a plastic such as polyethylene or Silly Putty® very slowly, the polymer molecules have time to disentangle or the chains to slide past each other and cause large plastic deformations. If, however, we apply an impact loading, there is insufficient time for these mechanisms to play a role and the materials break in a brittle manner. An **impact test** is often used to evaluate the brittleness of a material under these conditions. In contrast to the tensile test, in this test, the strain rates are much higher ($\dot{\varepsilon} \sim 10^3\ s^{-1}$).

Many test procedures have been devised, including the *Charpy* test and the *Izod* test (Figure 6-23). The Izod test is often used for plastic materials. The test specimen may be either notched or unnotched; V-notched specimens better measure the resistance of the material to crack propagation.

In the test, a heavy pendulum, starting at an elevation h_0, swings through its arc, strikes and breaks the specimen, and reaches a lower final elevation h_f. If we know the initial and final elevations of the pendulum, we can calculate the difference in potential energy. This difference is the **impact energy** absorbed by the specimen during failure. For the Charpy test, the energy is usually expressed in foot-pounds (ft · lb) or joules (J), where 1 ft · lb = 1.356 J. The results of the Izod test are expressed in units of ft · lb/in. or J/m. The ability of a material to withstand an impact blow is often referred to as the **impact toughness** of the material. As we mentioned before, in some situations, we consider the area

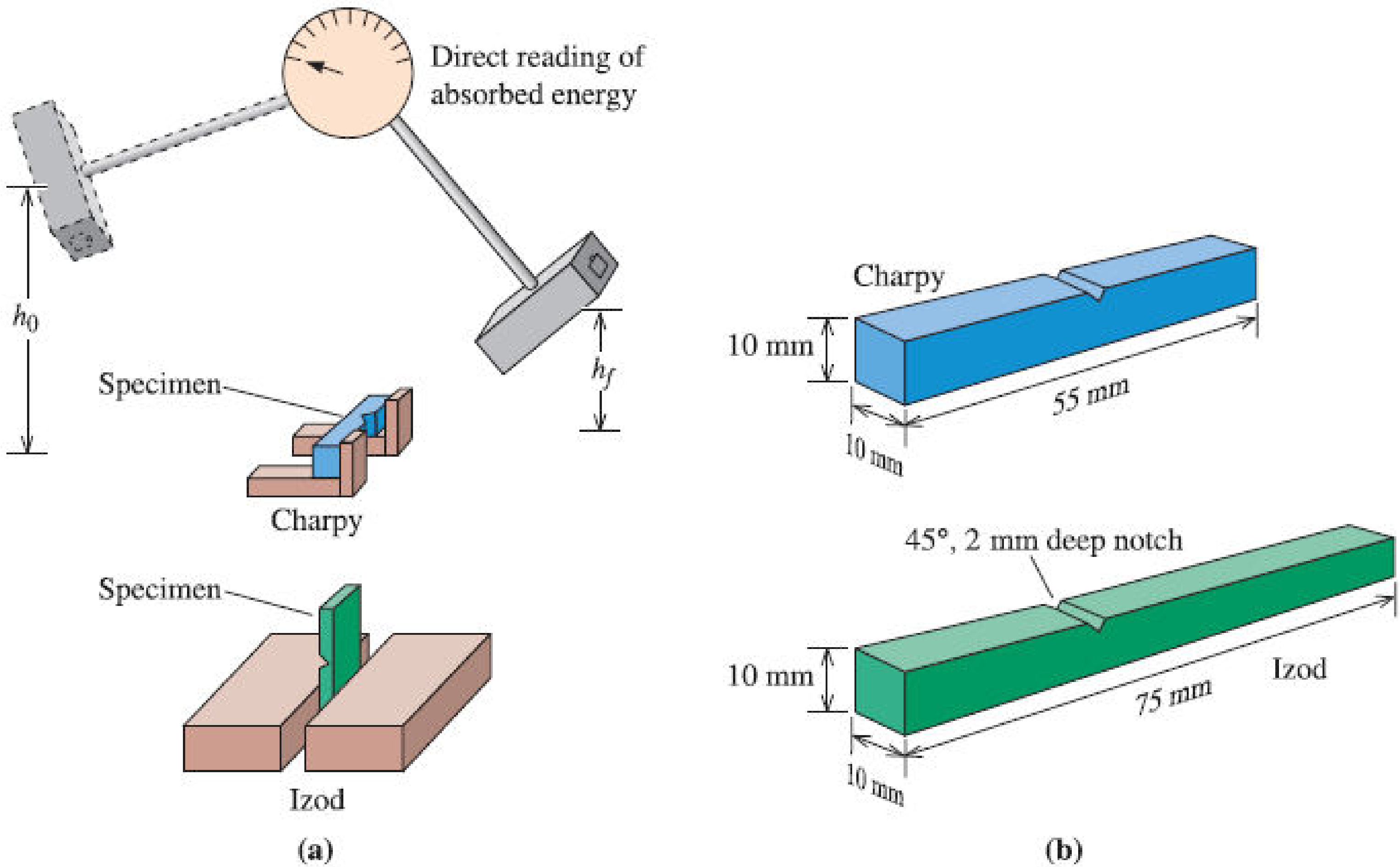


Figure 6-23 The impact test: (a) the Charpy and Izod tests, and (b) dimensions of typical specimens.

under the true or engineering stress-strain curve as a measure of **tensile toughness**. In both cases, we are measuring the energy needed to fracture a material. The difference is that, in tensile tests, the strain rates are much smaller compared to those used in an impact test. Another difference is that in an impact test we usually deal with materials that have a notch. **Fracture toughness** of a material is defined as the ability of a material containing flaws to withstand an applied load.

Ductile to Brittle Transition Temperature (DBTT) The **ductile to brittle transition temperature** is the temperature at which the failure mode of a material changes from ductile to brittle fracture. This temperature may be defined by the average energy between the ductile and brittle regions, at some specific absorbed energy, or by some characteristic fracture appearance. A material subjected to an impact blow during service should have a transition temperature *below* the temperature of the material's surroundings.

Not all materials have a distinct transition temperature (Figure 6-25). BCC metals have transition temperatures, but most FCC metals do not. FCC metals have high absorbed energies, with the energy decreasing gradually and, sometimes, even increasing as the temperature decreases. As mentioned before, the effect of this transition in steel may have contributed to the failure of the *Titanic*.

In polymeric materials, the ductile to brittle transition temperature is related closely to the glass-transition temperature and for practical purposes is treated as the same. As mentioned before, the transition temperature of the polymers used in booster rocket O-rings and other factors led to the *Challenger* disaster.

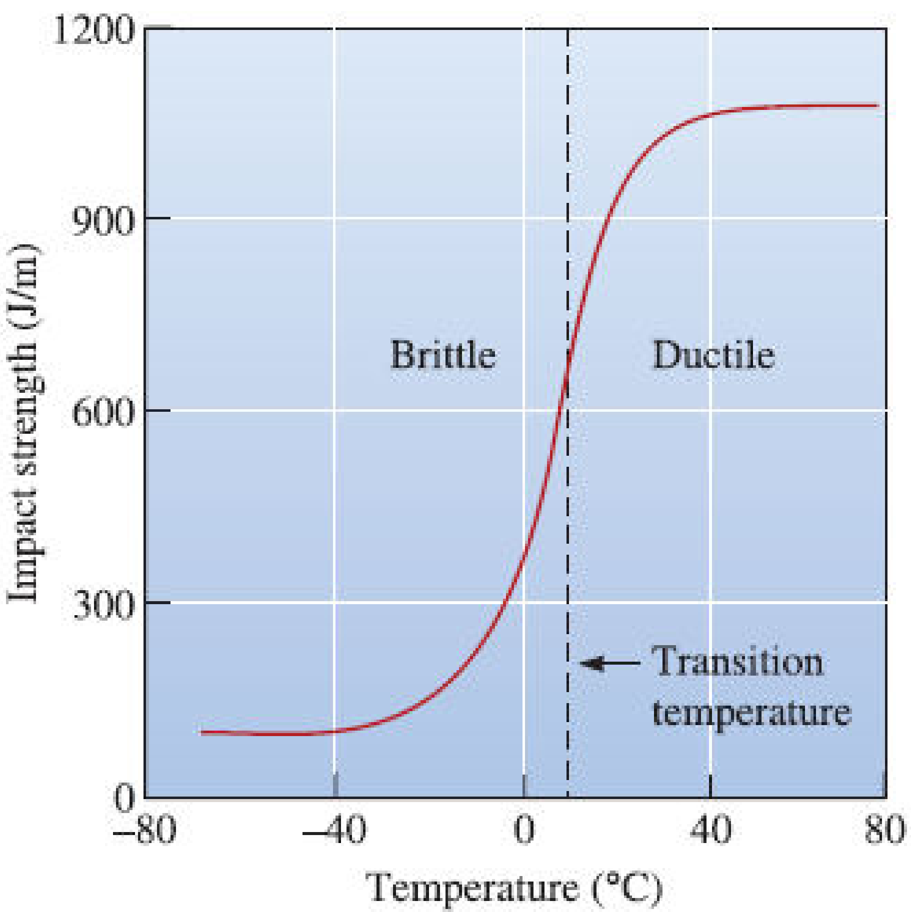


Figure 6-24
Results from a series of Izod impact tests for a tough nylon thermoplastic polymer.

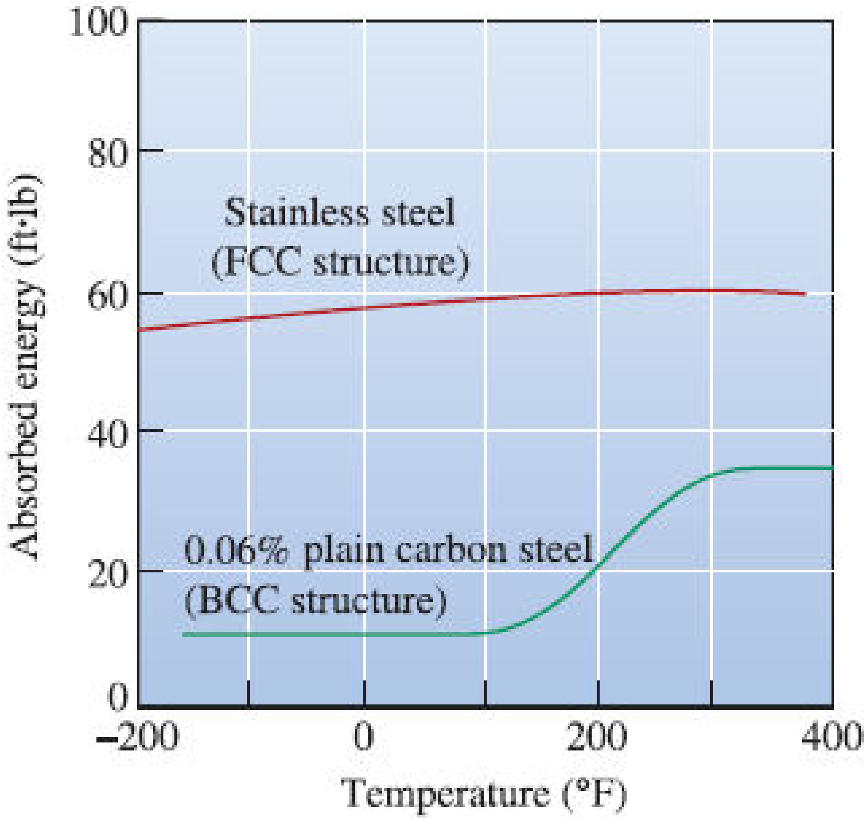


Figure 6-25
The Charpy V-notch properties for a BCC carbon steel and an FCC stainless steel. The FCC crystal structure typically leads to higher absorbed energies and no transition temperature.

Notch Sensitivity Notches caused by poor machining, fabrication, or design concentrate stresses and reduce the toughness of materials. The **notch sensitivity** of a material can be evaluated by comparing the absorbed energies of notched versus unnotched specimens. The absorbed energies are much lower in notched specimens if the material is notch-sensitive.

Relationship to the Stress-Strain Diagram The energy required to break a material during impact testing (i.e., the impact toughness) is not always related to the tensile toughness (i.e., the area contained under the true stress-true strain curve (Figure 6-26). As noted before, engineers often consider the area under the engineering stress–strain curve as tensile toughness. In general, metals with both high strength and high ductility have good tensile toughness; however, this is not always the case when the strain rates are high. For example, metals that show excellent tensile toughness may show brittle behavior under high strain rates (i.e., they may show poor impact toughness). Thus, the imposed strain rate can shift the ductile to brittle transition. Ceramics and many

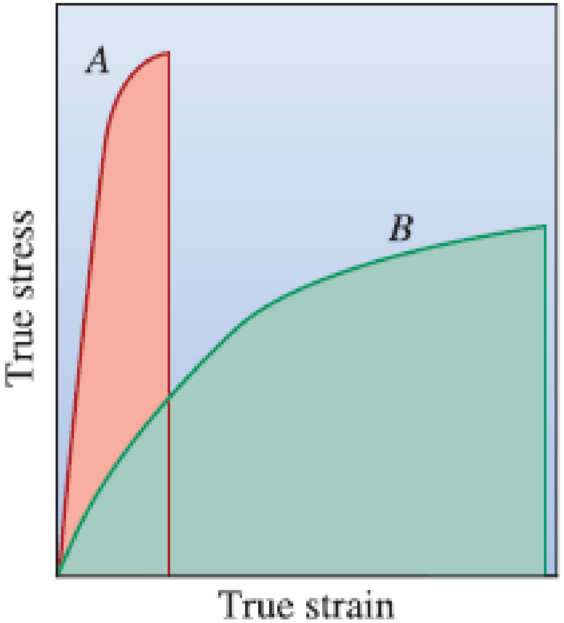


Figure 6-26
The area contained under the true stress-true strain curve is related to the tensile toughness. Although material *B* has a lower yield strength, it absorbs more energy than material *A*. The energies from these curves may not be the same as those obtained from impact test data.

composites normally have poor toughness, even though they have high strength, because they display virtually no ductility. These materials show both poor tensile toughness and poor impact toughness.

Use of Impact Properties Absorbed energy and the DBTT are very sensitive to loading conditions. For example, a higher rate of application of energy to the specimen reduces the absorbed energy and increases the DBTT. The size of the specimen also affects the results; because it is more difficult for a thick material to deform, smaller energies are required to break thicker materials. Finally, the configuration of the notch affects the behavior; a sharp, pointed surface crack permits lower absorbed energies than does a V-notch. Because we often cannot predict or control all of these conditions, the impact test is a quick, convenient, and inexpensive way to compare different materials.

Example 6-8 *Design of a Sledgehammer*

Design an eight pound sledgehammer for driving steel fence posts into the ground.

SOLUTION

First, we must consider the design requirements to be met by the sledgehammer. A partial list would include

1. The handle should be light in weight, yet tough enough that it will not catastrophically break.
2. The head must not break or chip during use, even in subzero temperatures.
3. The head must not deform during continued use.
4. The head must be large enough to ensure that the user does not miss the fence post, and it should not include sharp notches that might cause chipping.
5. The sledgehammer should be inexpensive.

Although the handle could be a lightweight, tough composite material (such as a polymer reinforced with Kevlar (a special polymer) fibers), a wood handle about 30 in. long would be much less expensive and would still provide sufficient toughness. As shown later in Chapter 17, wood can be categorized as a natural fiber-reinforced composite.

To produce the head, we prefer a material that has a low transition temperature, can absorb relatively high energy during impact, and yet also has enough hardness to avoid deformation. The toughness requirement would rule out most ceramics. A face-centered cubic metal, such as FCC stainless steel or copper, might provide superior toughness even at low temperatures; however, these metals are relatively soft and expensive. An appropriate choice might be a BCC steel. Ordinary steels are inexpensive, have good hardness and strength, and some have sufficient toughness at low temperatures.

In Appendix A, we find that the density of iron is 7.87 g/cm^3, or 0.28 $lb/in.^3$ We assume that the density of steel is about the same. The volume of steel required is $V = 8\ lbs/(0.28\ lb/in.) = 28.6\ in.^3$ To ensure that we will hit our target, the head might have a cylindrical shape with a diameter of 2.5 in. The length of the head would then be 5.8 in.

Fatigue

Fatigue is the lowering of strength or failure of a material due to repetitive stress which may be above or below the yield strength. It is a common phenomenon in load-bearing components in cars and airplanes, turbine blades, springs, crankshafts and other machinery, biomedical implants, and consumer products, such as shoes, that are subjected constantly to repetitive stresses in the form of tension, compression, bending, vibration, thermal expansion and contraction, or other stresses. These stresses are often *below* the yield strength of the material; however, when the stress occurs a sufficient number of times, it causes failure by fatigue! Quite a large fraction of components found in an automobile junkyard belongs to those that failed by fatigue. The possibility of a fatigue failure is the main reason why aircraft components have a finite life.

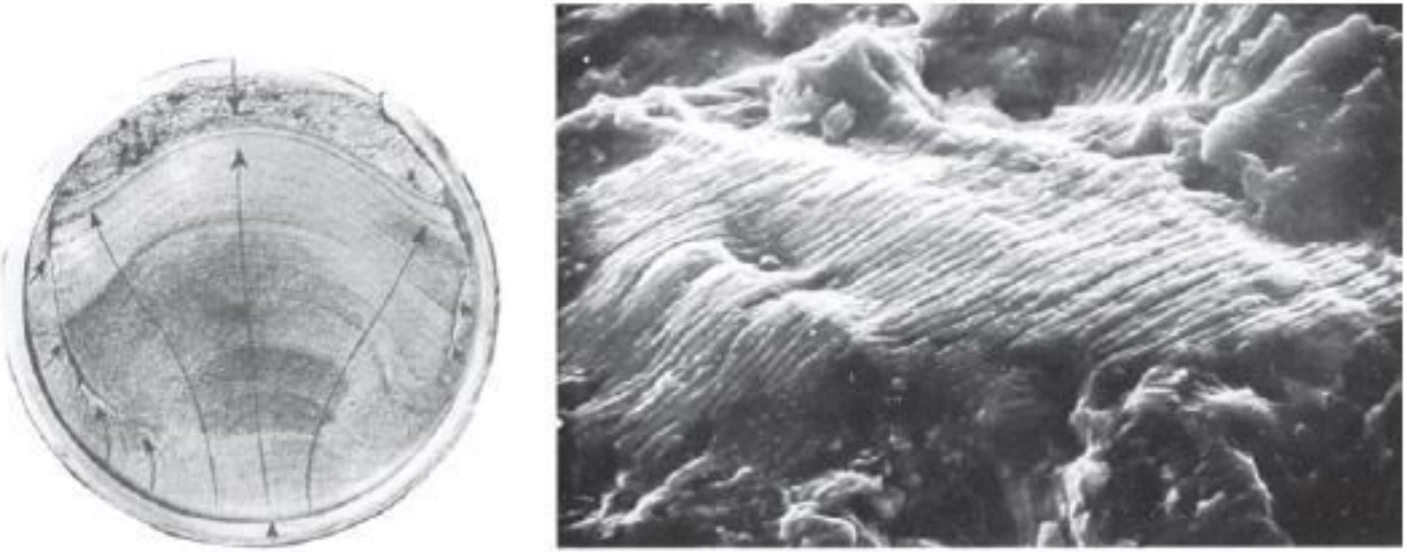
Fatigue failures typically occur in three stages. First, a tiny crack initiates or nucleates often at a time well after loading begins. Normally, nucleation sites are located at or near the surface, where the stress is at a maximum, and include surface defects such as scratches or pits, sharp corners due to poor design or manufacture, inclusions, grain boundaries, or dislocation concentrations. Next, the crack gradually propagates as the load continues to cycle. Finally, a sudden fracture of the material occurs when the remaining cross-section of the material is too small to support the applied load. Thus, components fail by fatigue because even though the overall applied stress may remain below the yield stress, at a local length scale, the stress intensity exceeds the tensile strength. For fatigue to occur, at least part of the stress in the material has to be tensile. We normally are concerned with fatigue of metallic and polymeric materials.

In ceramics, we normally do not consider fatigue since ceramics typically fail because of their low fracture toughness. Any fatigue cracks that may form will lower the useful life of the ceramic since it will cause lowering of the fracture toughness. In general, we design ceramics for static (and not cyclic) loading, and we factor in the Weibull modulus.

Polymeric materials also show fatigue failure. The mechanism of fatigue in polymers is different than that in metallic materials. In polymers, as the materials are subjected to repetitive stresses, considerable heating can occur near the crack tips and the interrelationships between fatigue and another mechanism, known as *creep* (discussed in Section 7-9), affect the overall behavior.

Fatigue is also important in dealing with composites. As fibers or other reinforcing phases begin to degrade as a result of fatigue, the overall elastic modulus of the composite decreases and this weakening will be seen before the fracture due to fatigue.

Fatigue failures are often easy to identify. The fracture surface—particularly near the origin—is typically smooth. The surface becomes rougher as the original crack increases in size and may be fibrous during final crack propagation. Microscopic and macroscopic examinations reveal a fracture surface including a beach mark pattern and striations (Figure 7-16). **Beach** or **clamshell marks** (Figure 7-17) are normally formed when the load is changed during service or when the loading is intermittent, perhaps permitting time for oxidation inside the crack. **Striations**, which are on a much finer scale, show the position of the crack tip after each cycle. Beach marks always suggest a fatigue failure, but—unfortunately—the absence of beach marks does not rule out fatigue failure.



(a) (b)

Figure 7-16 Fatigue fracture surface. (a) At low magnifications, the beach mark pattern indicates fatigue as the fracture mechanism. The arrows show the direction of growth of the crack front with the origin at the bottom of the photograph. *(Image* (a) is *from C. C. Cottell,* "Fatigue Failures with Special Reference to Fracture Characteristics," Failure Analysis: The British Engine Technical Reports, *American Society for Metals, 1981, p. 318.)* (b) At very high magnifications, closely spaced striations formed during fatigue are observed (× 1000). (*Reprinted courtesy of Don Askeland.*)

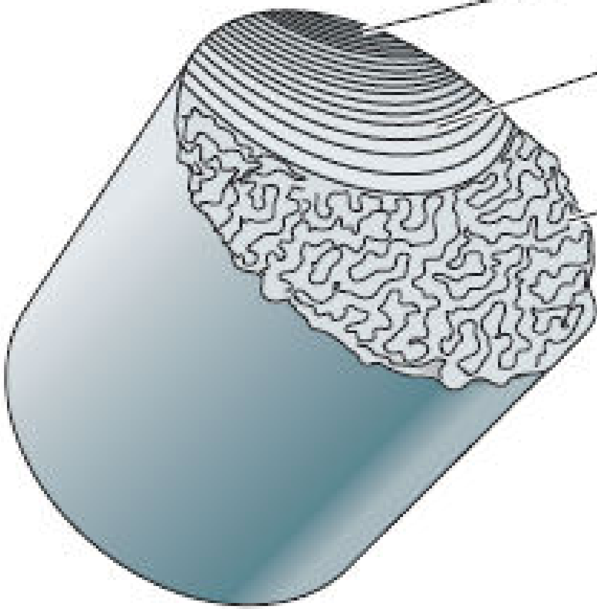


Figure 7-17
Schematic representation of a fatigue fracture surface in a steel shaft, showing the initiation region, the propagation of the fatigue crack (with beach markings), and catastrophic rupture when the crack length exceeds a critical value at the applied stress.

Results of the Fatigue Test

The **fatigue test** can tell us how long a part may survive or the maximum allowable loads that can be applied without causing failure. The **endurance limit**, which is the stress below which there is a 50% probability that failure by fatigue will never occur, is our preferred design criterion. To prevent a tool steel part from failing (Figure 7-19), we must be sure that the applied stress is below 60,000 psi. The assumption of the existence of an endurance limit is a relatively older concept. Recent research on many metals has shown that probably an endurance limit does not exist. We also need to account for the presence of corrosion, occasional overloads, and other mechanisms that may cause the material to fail below the endurance limit. Thus, values for an endurance limit should be treated with caution.

Fatigue life tells us how long a component survives at a particular stress. For example, if the tool steel (Figure 7-19) is cyclically subjected to an applied stress of 90,000 psi, the fatigue life will be 100,000 cycles. Knowing the time associated with each cycle, we can calculate a fatigue life value in years. **Fatigue strength** is the maximum stress for which fatigue will not occur within a particular number of cycles, such as 500,000,000. The fatigue strength is necessary for designing with aluminum and polymers, which have no endurance limit.

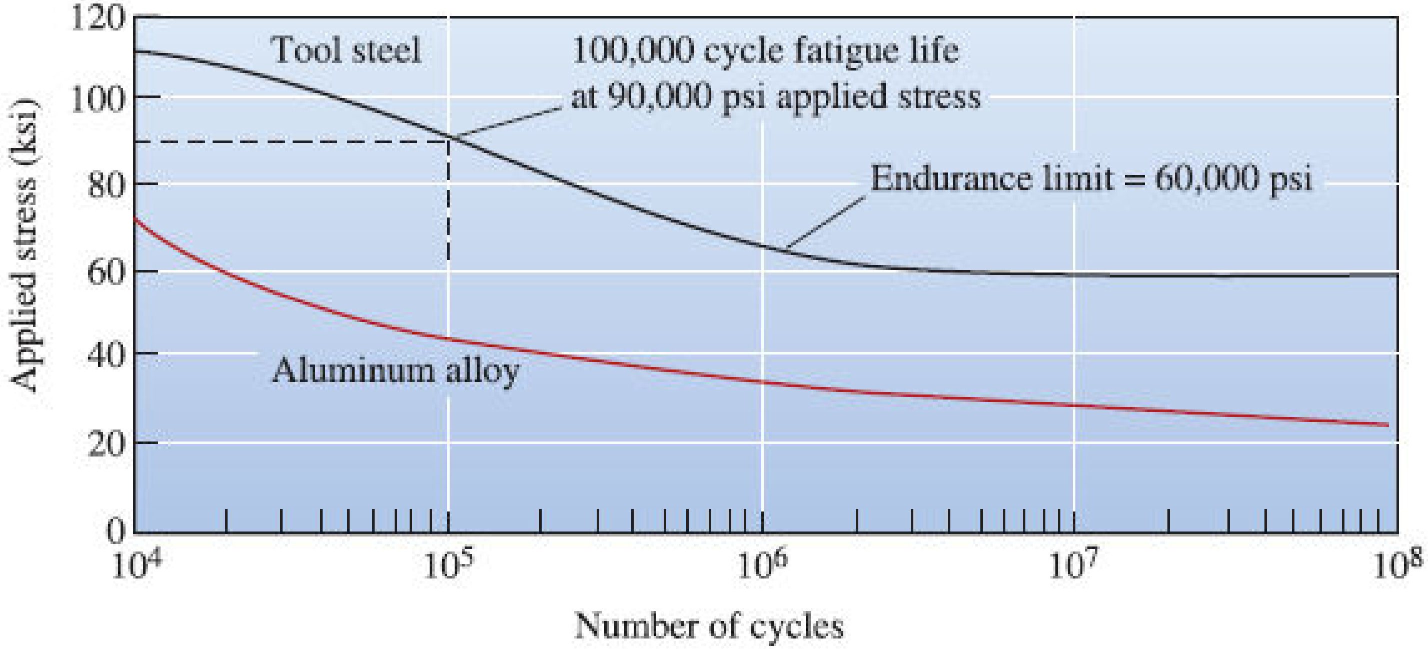


Figure 7-19 The stress-number of cycles to failure (S-N) curves for a tool steel and an aluminum alloy.

In some materials, including steels, the endurance limit is approximately half the tensile strength. The ratio between the endurance limit and the tensile strength is known as the **endurance ratio**:

$$\text{Endurance ratio} = \frac{\text{endurance limit}}{\text{tensile strength}} \approx 0.5 \qquad (7\text{-}14)$$

The endurance ratio allows us to estimate fatigue properties from the tensile test. The endurance ratio values are ~0.3 to 0.4 for metallic materials other than low and medium strength steels. *Again, recall the cautionary note that research has shown that an endurance limit does not exist for many materials.*

Most materials are **notch sensitive**, with the fatigue properties particularly sensitive to flaws at the surface. Design or manufacturing defects concentrate stresses and reduce the endurance limit, fatigue strength, or fatigue life. Sometimes highly polished surfaces are prepared in order to minimize the likelihood of a fatigue failure. **Shot peening** is a process that is used very effectively to enhance fatigue life of materials. Small metal spheres are shot at the component. This leads to a residual compressive stress at the surface similar to **tempering** of inorganic glasses.

Example 7-11 *Design of a Rotating Shaft*

A solid shaft for a cement kiln produced from the tool steel in Figure 7-19 must be 96 in. long and must survive continuous operation for one year with an applied load of 12,500 lb. The shaft makes one revolution per minute during operation. Design a shaft that will satisfy these requirements.

SOLUTION

The fatigue life required for our design is the total number of cycles N that the shaft will experience in one year:

$$N = (1 \text{ cycle/min})(60 \text{ min}/h)(24\ h/d)(365\ d/y).$$

$$N = 5.256 \times 10^5 \text{ cycles}/y$$

where y = year, d = day, and h = hour.

From Figure 7-19, the applied stress therefore, must be less than about 72,000 psi. Using Equation 7-13, the diameter of the shaft is given by

$$\pm\sigma = \frac{16FL}{\pi d^3} = 5.09\frac{FL}{d^3}$$

$$72{,}000 \text{ psi} = \frac{(5.09)(12{,}500 \text{ lb})(96 \text{ in.})}{d^3}$$

$$d = 4.39 \text{ in.}$$

A shaft with a diameter of 4.39 in. should operate for one year under these conditions; however, a significant margin of safety probably should be incorporated in the design. In addition, we might consider producing a shaft that would never fail.

Let us assume the factor of safety to be 2 (i.e., we will assume that the maximum allowed stress level will be 72,000/2 = 36,000 psi). The minimum diameter required to prevent failure would now be

$$36{,}000 \text{ psi} = \frac{(5.09)(12{,}500 \text{ lb})(96 \text{ in.})}{d^3}$$

$$d = 5.54 \text{ in.}$$

Selection of a larger shaft reduces the stress level and makes fatigue less likely to occur or delays the failure. Other considerations might, of course, be important. High temperatures and corrosive conditions are inherent in producing cement. If the shaft is heated or attacked by the corrosive environment, fatigue is accelerated. Thus, for applications involving fatigue of components, regular inspections of the components go a long way toward avoiding a catastrophic failure.

Application of Fatigue Testing

Components are often subjected to loading conditions that do not give equal stresses in tension and compression (Figure 7-20). For, example, the maximum stress during compression may be less than the maximum tensile stress. In other cases, the loading may be between a maximum and a minimum tensile stress; here the S-N curve is presented as the stress amplitude versus the number of cycles to failure. *Stress amplitude* (σ_a) is defined as half of the difference between the maximum and minimum stresses, and *mean stress* (σ_m) is defined as the average between the maximum and minimum stresses:

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2} \tag{7-15}$$

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} \tag{7-16}$$

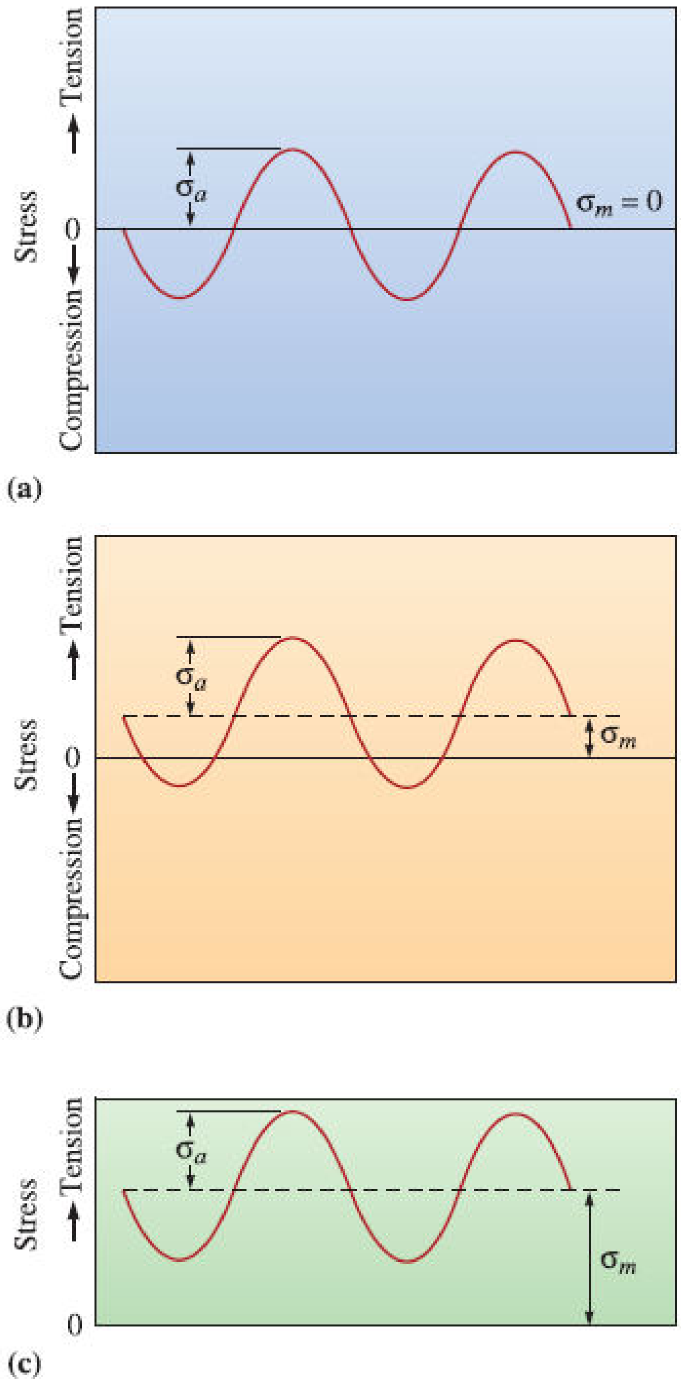


Figure 7-20
Examples of stress cycles. (a) Equal stress in tension and compression, (b) greater tensile stress than compressive stress, and (c) all of the stress is tensile.

A compressive stress is a "negative" stress. Thus, if the maximum tensile stress is 50,000 psi and the minimum stress is a 10,000 psi compressive stress, using Equations 7-15 and 7-16, the stress amplitude is 30,000 psi, and the mean stress is 20,000 psi.

As the mean stress increases, the stress amplitude must decrease in order for the material to withstand the applied stresses. The condition can be summarized by the Goodman relationship:

$$\sigma_a = \sigma_{fs}\left[1 - \left(\frac{\sigma_m}{\sigma_{UTS}}\right)\right] \tag{7-17}$$

where σ_{fs} is the desired fatigue strength for zero mean stress and σ_{UTS} is the tensile strength of the material. Therefore, in a typical rotating cantilever beam fatigue test, where the mean stress is zero, a relatively large stress amplitude can be tolerated without fatigue. If, however, an airplane wing is loaded near its yield strength, vibrations of even a small amplitude may cause a fatigue crack to initiate and grow.

Example 7-12 *Design of a Fatigue Resistant Plate*

A high-strength steel plate (Figure 7-21), which has a plane strain fracture toughness of 80 MPa$\sqrt{\text{m}}$ is alternately loaded in tension to 500 MPa and in compression to 60 MPa. The plate is to survive for 10 years with the stress being applied at a frequency of once every 5 minutes. Design a manufacturing and testing procedure that ensures that the component will serve as intended. Assume a geometry factor $f = 1.0$ for all flaws.

SOLUTION

To design our manufacturing and testing capability, we must determine the maximum size of any flaws that might lead to failure within the 10 year period. The critical crack size using the fracture toughness and the maximum stress is

$$K_{Ic} = f\sigma\sqrt{\pi a_c}$$

$$80 \text{ MPa}\sqrt{\text{m}} = (1.0)(500 \text{ MPa})\sqrt{\pi a_c}$$

$$a_c = 0.0081 \text{ m} = 8.1 \text{ mm}$$

The maximum stress is 500 MPa; however, the minimum stress is zero, not 60 MPa in compression, because cracks do not propagate in compression. Thus, $\Delta\sigma$ is

$$\Delta\sigma = \sigma_{\text{max}} - \sigma_{\text{min}} = 500 - 0 = 500 \text{ MPa}$$

We need to determine the minimum number of cycles that the plate must withstand:

$$N = (1\ \text{cycle}/5\ \text{min})(60\ \text{min}/h)(24\ h/d)(365\ d/y)(10\ y)$$

$$N = 1{,}051{,}200\ \text{cycles}$$

If we assume that $f = 1.0$ for all crack lengths and note that $C = 1.62 \times 10^{-12}$ and $n = 3.2$ from Figure 7-21 in Equation 7-20, then

$$1{,}051{,}200 = \frac{2\left[(0.008)^{(2-3.2)/2} - (a_i)^{(2-3.2)/2}\right]}{(2 - 3.2)(1.62 \times 10^{-12})(1)^{3.2}(500)^{3.2}\pi^{3.2/2}}$$

$$1{,}051{,}200 = \frac{2\left[18 - a_i^{0.6}\right]}{(-1.2)(1.62 \times 10^{-12})(1)(4.332 \times 10^{8})(6.244)}$$

$$a_i^{-0.6} = 18 + 2764 = 2782$$

$$a_i = 1.82 \times 10^{-6}\ \text{m} = 0.00182\ \text{mm for surface flaws}$$

$$2a_i = 0.00364\ \text{mm for internal flaws}$$

The manufacturing process must produce surface flaws smaller than 0.00182 mm in length. In addition, nondestructive tests must be available to ensure that cracks approaching this length are not present.

Thank you for your attention